Initial State Correlations and the Ridge

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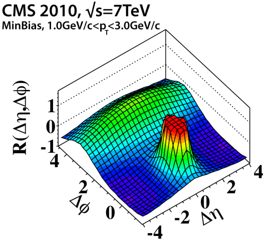
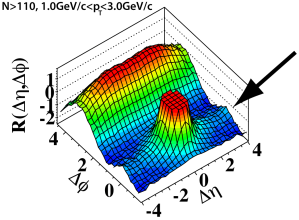
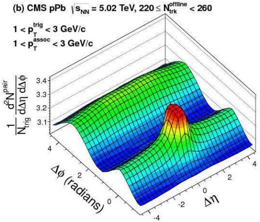
Phys.Lett. B751 (2015), Phys.Lett. B752 (2016), arXiv:1610.XXXXX

The Ridge Structure







Correlations within the CGC

Final state correlations carry the imprint of the partonic correlations that exist in the initial state:

1. Local anisotropy of target fields
   - A. Kovner, M. Lublinsky, Phys.Rev. D83 (2011) 034017
   - A. Kovner, M. Lublinsky, Phys.Rev. D84 (2011) 094011
   - A. Kovner, M. Lublinsky, Int. J. Mod. Phys. E Vol. 22 (2013) 1330001
   - A. Dumitru, L. McLerran, V. Skokov, Phys. Lett. B 743 (2015)
   - A. Dumitru, V. Skokov, Phys.Rev. D91 (2015) 074006
2. Spatial variation of partonic density
   - E. Levin, A. Rezaeian, Phys.Rev. D84 (2011) 034031
3. "Glasma Graph" contributions to particle production:
   - A. Dumitru, F. Gelis, J. Jalilian-Marian, T. Lappi, Phys.Lett. B697 (2011) 21
   - K. Dusling, R. Venugopalan, Phys.Rev.Lett. 108 (2012) 262001
   - K. Dusling, R. Venugopalan, Phys.Rev. D87 (2013) 051502
   - K. Dusling, R. Venugopalan, Phys.Rev. D87 (2013) 054014

WHAT IS THE PHYSICS BEHIND THE GLASMA GRAPH CALCULATIONS?

$\Downarrow$

BOSE ENHANCEMENT OF GLUONS IN THE HADRONIC WAVE-FUNCTION!!!

Bose enhancement in a nutshell (1)

Consider a state with fixed occupation numbers of N species of bosons at different momenta:

$$\boxed{\left|\{n^i(p)\}\right\rangle \equiv \prod_{i,p} \frac{1}{\sqrt{n^i(p)!}} \left(\frac{a_i^\dagger(p)}{\sqrt{V}}\right)^{n^i(p)} |0\rangle}$$

with a finite volume V and periodic boundary conditions so that momenta are discrete. $(i = 1, 2, \cdots, N)$

The mean particle density :

$$\boxed{n \equiv \left\langle \{n^i(p)\} | a^{\dagger i}(x) a^i(x) | \{n^i(p)\} \right\rangle = \sum_{i,p} n^i(p)}$$

The 2-particle correlator:

in x-space $\rightarrow$ $\boxed{D(x,y) \equiv \langle \{n(p)\} | a^{\dagger i}(x) a^{\dagger j}(y) a^i(x) a^j(y) | \{n(p)\} \rangle}$

in p-space $\rightarrow$ $\boxed{D(p,k) \equiv \langle \{n(p)\} | a^{\dagger i}(p) a^{\dagger j}(q) a^i(l) a^j(m) | \{n(p)\} \rangle}$

$$\Rightarrow D(p,k) = \delta(p-l)\delta(q-m) \sum_i n^i(p) \sum_j n^j(q) + \delta(p-m)\delta(q-l) \sum_i n^i(p) n^i(q)$$

Bose enhancement in a nutshell (2)

Using these results, the 2-particle correlator in coordinate space:

$$D(x,y) = n^2 + \underbrace{\sum_i \left| \int \frac{d^3p}{(2\pi)^3} e^{ip(x-y)} n^i(p) \right|^2}_{\text{the Bose enhancement term}}$$

in momentum space:

$$D(p,k) = \left[\sum_i n_i(p)\right] \left[\sum_j n_j(k)\right] + \overbrace{\delta(p-k)\sum_i [n_i(p)]^2}$$

- It vanishes when the points are far away!
- It gives $O(1/N)$ enhancement when the points coincide!

The $O(1/N)$ suppression is due to the fact the second term contains a single sum over the species index!

The physics: **Only bosons of the same species are correlated with each other.**

Bose enhancement in CGC?

The Bose enhancement is a generic phenomenon, and is not tied to the state with fixed number of of particles.
HOWEVER,
Classical-like coherent states do not exhibit such behaviour!!!
Consider a coherent state:

$$|b(x)\rangle \equiv \exp\left\{ i \int d^3x \, b^i(x) \left[a^i(x) + a^{\dagger i}(x) \right] \right\} |0\rangle$$

A trivial calculation in this state gives

$$\langle b(x)|a^{\dagger i}(x)a^i(x)|b(x)\rangle = b^i(x)b^i(x)$$
$$\langle b(x)|a^{\dagger i}(x)a^{\dagger j}(y)a^i(x)a^j(y)|b(x)\rangle = b^i(x)b^i(x)b^j(y)b^j(y)$$

so

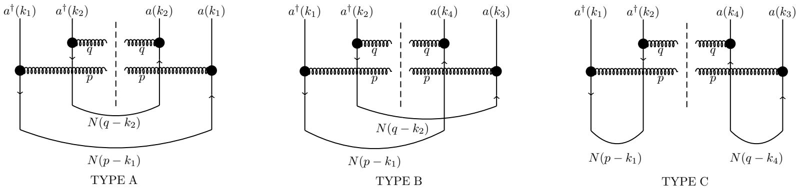
$$\boxed{D(x, y) = n(x)n(y)}$$

Thus, in order to exhibit Bose enhancement, a state has to be nonclassical.
CGC is defined in terms of classical fields. Can they produce Bose enhancement??
YES!!

Glasma Graphs

Aim: **to show that angular collimation arising from glasma graph calculation is due to the Bose enhancement in the projectile wave-function.**

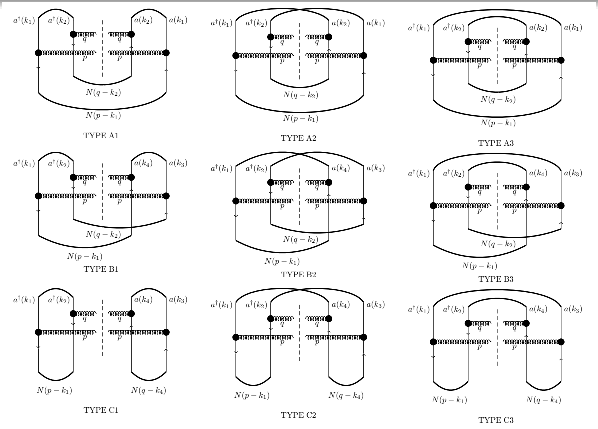
Consider inclusive two particle production and assume parton-hadron duality.



with $N(k)$ $\equiv$dipole scattering probability :

$$N(k) = -\int d^2x\, e^{i\mathbf{k}\cdot\mathbf{x}} \left\langle \frac{1}{N_c} tr\left[S^\dagger(x)S(0)\right]\right\rangle_T$$

Averaging over the projectile



- A3 is trivial contraction! (not interesting!)
- B3 & C3 $\rightarrow$ A1 & A2 with T $\leftrightarrow$ P.
- B1 & C2: suppressed at high momenta.

- B2 & C1:
  - suppressed at high momenta
  - $\propto \delta^{(2)}(p \pm q)$
  - lead to HBT correlations [Kovchegov, Wertepny] (2013)

Gluon Production

$$\text{Type A} \propto \int_{k_1,k_2} \underbrace{\langle in|a_a^{\dagger i}(k_1)a_b^{\dagger j}(k_2)a_a^k(k_1)a_b^l(k_2)|in\rangle_P}_{D(k_1,k_2)} N(p-k_1)\, N(q-k_2)$$

$N(p-k) \equiv$ the probability that incoming gluon with transverse momentum k acquires transverse momentum p after the scattering.

CGC hadronic wave-function is boost invariant.

$$\Downarrow$$

$$a_a^i(k) \equiv \frac{1}{\sqrt{Y}} \int_{|\eta<Y/2|} \frac{d\eta}{2\pi}\; a_a^i(\eta,k).$$

with the standard commutation relations:

$$\boxed{[a_a^i(k), a_b^{\dagger j}(p)] = (2\pi)^2 \delta_{ab}\delta^{ij}\delta^{(2)}(k-p).}$$

Averaging over the projectile

Averaging over the projectile state in CGC:

- average over soft degrees of freedom
- average over the valence color charge density ρ

The wave-function for the soft fields at fixed ρ:

$$|in\rangle_\rho = \exp\left\{ i \int_k b_a^i(k) \left[a_a^{\dagger i}(k) + a_a^i(-k) \right] \right\} |0\rangle$$

Weizsäcker-Williams field $b_a^i(k) = g\rho_a(k)\frac{ik^i}{k^2}$

Averaging over $\rho \Rightarrow$ integrating over ρ with some weight functional $W[\rho]$!

Take MV model: $\langle \cdots \rangle_\rho = \mathcal{N} \int D[\rho] \cdots e^{-\int_k \frac{1}{2\mu^2(k)} \rho_a(k)\rho_a(-k)}$

This defines the density matrix (operator) on the soft gluon Hilbert space:

$$\boxed{\hat{\rho} = \mathcal{N} \int D[\rho]\, e^{-\int_k \frac{1}{2\mu^2(k)} \rho_a(k)\rho_a(-k)} e^{i\int_q b_b^i(q)\phi_b^i(-q)} |0\rangle\langle 0|\, e^{-i\int_p b_c^j(p)\phi_c^j(-p)}}$$

where $\phi_a^i(k) = a_a^i(k) + a_a^{\dagger i}(-k)$.

Integration over ρ gives:

$$\hat{\rho} = e^{-\int_k \frac{g^2\mu^2(k)}{2k^4} k^i k^j \phi^i_b(k)\,\phi^j_b(-k)} \left\{ \sum_{n=0}^{+\infty} \frac{1}{n!} \left[\prod_{m=1}^{n} \int_{p_m} \frac{g^2\mu^2(p_m)}{p_m^4}\, p_m^{i_m}\, \phi^{i_m}_{a_m}(p_m) \right] |0\rangle \right.$$
$$\times \left. \langle 0| \left[\prod_{m=1}^{n} p_m^{j_m} \phi^{j_m}_{a_m}(-p_m) \right] \right\} e^{-\int_{k'} \frac{g^2\mu^2(k')}{2k'^4} k'^{i'} k'^{j'}\, \phi^{i'}_c(k')\,\phi^{j'}_c(-k')}$$

The correlator that we are interested in :

$$\boxed{D(k_1,k_2) = tr[\hat{\rho} a^{\dagger i}_a(k_1) a^{\dagger j}_b(k_2) a^k_a(k_1) a^l_b(k_2)]}$$

$$D(k_1,k_2) = S^2(N_c^2-1)^2 \frac{k_1^i k_1^k k_2^j k_2^l}{k_1^2 k_2^2} \frac{g^4\mu^2(k_1)\mu^2(k_2)}{k_1^2 k_2^2}$$
$$\times \left\{ 1 + \frac{1}{S(N_c^2-1)} \left[\delta^{(2)}(k_1-k_2) + \delta^{(2)}(k_1+k_2) \right] \right\}.$$

The first term is the "classical" term (the square of the number of particles)
The second term is the typical Bose enhancement term!

Correlated production

Consider:

- Incoming projectile wave-function with saturation momentum Q_s
- choose k_1 and k_2 to be of the same order as Q_s

Then:

- the momentum transfer during the scattering is less than Q_s
- $N(p-k_1)N(p-k_2)$ does not have large effect!

Thus the initial correlation is transmitted to to final state.

FOR GLUONS:

INITIAL STATE BOSE ENHANCEMENT $\Rightarrow$ FINAL STATE CORRELATIONS

What about quark correlations: Pauli blocking?

Are quarks in the CGC state subject to correlations?

Within the "glasma graph" approach:

- gluons:
  - final state correlations are due to the Bose enhancement of the gluons in the projectile wave function.
  - This effect is long range in rapidity since the CGC wave function is dominated by the rapidity integrated mode of the soft gluon field.
- quarks:
  - quarks should experience Pauli blocking ⇒ the probability to find two identical quarks with the same quantum numbers in the CGC state should be supressed.
  - Is the effect short or long range in rapidity?

Quark contribution to the wave function (1)

The free part of the Light Cone Hamiltonian:

$$H_0 = \int_{k^+,k} \frac{k^2}{2k^+}\; a_i^{\dagger a}(k^+,k)\; a_i^a(k^+,k)$$
$$+\sum_s \int_{p^+,p} \frac{p^2}{2p^+}\left[d^\dagger_{\alpha\, s}(p^+,p)\, d_{\alpha\, s}(p^+,p) + \bar{d}^\dagger_{\alpha\, s}(p^+,p)\, \bar{d}_{\alpha\, s}(p^+,p)\right]$$

To zeroth order the vacuum of the LCH is simply the zero energy Fock space vacuum of the operators a, d and $\bar{d}$:

$$a_q|0\rangle = 0, \quad d_p|0\rangle = 0, \quad \bar{d}_p|0\rangle = 0, \quad E_0 = 0.$$

The normalized one-particle states to zeroth order are

$$\begin{aligned}
|k^+, k, a, i\rangle &= \frac{1}{(2\pi)^{3/2}}\, a_i^{a\,\dagger}(k^+,k)\,|0\rangle, \\
\langle k_1^+, k_1, a, i|k_2^+, k_2, b, j\rangle &= \delta^{ab}\,\delta_{ij}\,\delta^{(2)}(k_1-k_2)\delta(k_1^+-k_2^+), \\
|p^+, p, \alpha, s\rangle &= \frac{1}{(2\,\pi)^{3/2}}\, d^\dagger_{\alpha,s}(p^+,p)\,|0\rangle, \\
\langle p_1^+, p_1, \alpha, s_1|p_2^+, p_2, \beta, s_2\rangle &= \delta_{\alpha\beta}\,\delta_{s_1 s_2}\,\delta^{(2)}(p_1-p_2)\delta(p_1^+-p_2^+)
\end{aligned}$$

Quark contribution to the wave function (2)

The full Hamiltonian contains several types of perturbations:

$$\delta H = \delta H^{\rho} + \delta H^{g\,qq} + \cdots$$

Interaction with the background field: $\delta H^{\rho} = \delta H^{\rho\,g} + \delta H^{\rho\,qq} + \delta H^{\rho\,gg}$

Quark-gluon interaction: $\delta H^{g\,qq}$

- using the explicit expressions of the corrections to the free Hamiltonian, we can write the relevant matrix elements needed for the calculation of the dressed quark wave function:

$$\langle g|\delta H^{\rho\,g}|0\rangle, \quad \langle q\bar{q}|\delta H^{\rho qq}|0\rangle, \quad \langle q\bar{q}|\delta H^{g\,qq}|g\rangle$$

- with all these ingredients the dressed wave function

$$|v\rangle_4^D = g^4 \int_{\underline{p},\underline{\bar{p}},\underline{q},\underline{\bar{q}}} \Big[\zeta^{\epsilon\iota}_{s_1 s_2}(k^+, p, \bar{p}, \alpha)\zeta^{\gamma\delta}_{r_1 r_2}(\bar{k}^+, q, \bar{q}, \beta)\; d^{\dagger}_{\epsilon,s_1}(\underline{p})\, \bar{d}^{\dagger}_{\iota,s_2}(\underline{\bar{p}})\; d^{\dagger}_{\gamma,r_1}(\underline{q})\, \bar{d}^{\dagger}_{\delta,r_2}(\underline{\bar{q}})\Big]\, |v\rangle$$
$$+\text{virtual}$$

The amplitude $\zeta^{\gamma\delta}_{s_1 s_2}(k^+, p, q, \alpha) = \frac{\tau^a_{\gamma\delta}}{k^+} \int_k \rho^a(k)\; \phi_{s_1 s_2}(k, p, q; \alpha)$

Quark pair production cross section

The formal expression for the inclusive quark pair production emission reads

$$\frac{d\sigma}{d\eta_1 d^2 k_1 d\eta_2 d^2 k_2} = \langle 0|\Omega\, \hat{S}^\dagger\, \Omega^\dagger d^\dagger_{\alpha,s_1}(\underline{k_1})\, d_{\alpha,s_1}(\underline{k_1})\, d^\dagger_{\beta,s_2}(\underline{k_2})\, d_{\beta,s_2}(\underline{k_2})\, \Omega\, \hat{S}\, \Omega^\dagger|0\rangle$$

- $\hat{S}$: eikonal S-matrix operator
- Ω: unitary operator that (perturbatively) diagonalizes the QCD Hamiltonian.

The quark pair production cross section can be written as

$$\begin{aligned}\frac{d\sigma}{d\eta_1\, d^2p\, d\eta_2\, d^2q} = \frac{g^8}{(2\pi)^6}\int \frac{1}{2}\, &\langle\rho^a(x)\rho^b(\bar{x})\rho^c(y)\rho^d(\bar{y})\rangle\Big[\Phi_2(x,y;z_1,z_2,\bar{z};p)\Phi_2(\bar{x},\bar{y};\bar{z}_1,\bar{z}_2,\bar{w};q)\\
&\times \mathrm{tr}\Big\{[\tau^a - S_A^{a\bar{a}}(x)S_F(z_1)\tau^{\bar{a}}S_F^\dagger(\bar{z})][\tau^c - S_A^{c\bar{c}}(y)S_F(\bar{z})\tau^{\bar{c}}S_F^\dagger(z_2)]\Big\}\\
&\times \mathrm{tr}\Big\{[\tau^b - S_A^{b\bar{b}}(\bar{x})S_F(\bar{z}_1)\tau^{\bar{b}}S_F^\dagger(\bar{w})][\tau^d - S_A^{d\bar{d}}(\bar{y})S_F(\bar{w})\tau^{\bar{d}}S_F^\dagger(\bar{z}_2)]\Big\}\\
&-\Phi_4(x,y,\bar{x},\bar{y};z_1,z_2,\bar{z}_1,\bar{z}_2;\bar{z},\bar{w};p,q)\\
&\times \mathrm{tr}\Big\{[\tau^a - S_A^{a\bar{a}}(x)S_F(z_1)\tau^{\bar{a}}S_F^\dagger(\bar{z})][\tau^c - S_A^{c\bar{c}}(y)S_F(\bar{w})\tau^{\bar{c}}S_F^\dagger(z_2)]\\
&\times\ [\tau^b - S_A^{b\bar{b}}(\bar{x})S_F(\bar{z}_1)\tau^{\bar{b}}S_F^\dagger(\bar{w})][\tau^d - S_A^{d\bar{d}}(\bar{y})S_F(\bar{z})\tau^{\bar{d}}S_F^\dagger(\bar{z}_2)]\Big\}\Big]\end{aligned}$$

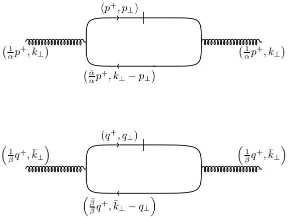
To get the quark pair density set each S-matrix to zero!

The amplitudes:

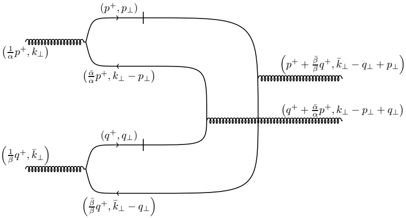
$$\Phi_2(k,p) \equiv \int_0^1 d\alpha \int_q \sum_{s_1\, s_2} \phi_{s_1,s_2}(k,p,q;\alpha)\, \phi^*_{s_1,s_2}(k,p,q;\alpha)$$

$$\Phi_4(k,l,\bar{k},\bar{l};p,q) \equiv \sum_{s_1\, s_2,\bar{s}_1,\bar{s}_2} \int_0^1 \frac{d\alpha\, d\beta}{(\beta+\bar{\beta}e^{\eta_1-\eta_2})(\alpha+\bar{\alpha}e^{\eta_2-\eta_1})}$$
$$\times \int_{\bar{p}\bar{q}} \phi_{s_1 s_2}(k,p,\bar{p};\alpha)\, \phi_{\bar{s}_1\bar{s}_2}(\bar{k},q,\bar{q};\beta)\, \phi^*_{s_1\bar{s}_2}(l,p,\bar{q};\beta)\, \phi^*_{\bar{s}_1 s_2}(\bar{l},q,\bar{p};\alpha)$$

Two different contributions to the wave function:

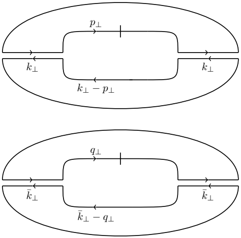


$\Phi_2\Phi_2$

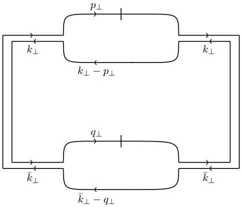


Φ_4

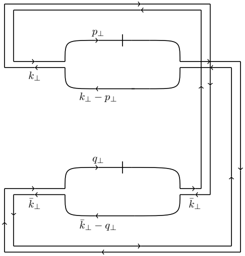
$\Phi_2\Phi_2$ contribution after contracting the color charge densities:



Uncorrelated $O(N_c^4)$



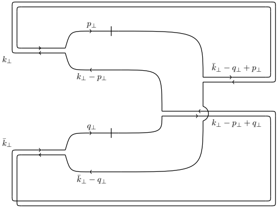
Correlated $O(N_c^2)$



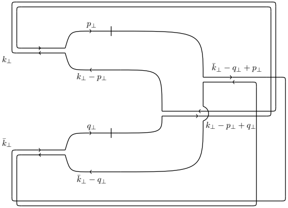
Correlated $O(N_c^2)$

$\Phi_2\Phi_2$ contribution to the correlated quark pair density is $O(N_c^2)$.

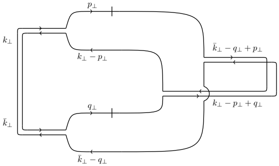
Φ_4 contribution after contracting the color charge densities:



(Φ_4^A) Correlated $O(N_c^3)$



(Φ_4^B) Correlated $O(N_c^3)$



Correlated $O(N_c)$

Φ_4 contribution to the correlated quark pair density is $O(N_c^3)$.

In the large N_c-limit, the only contribution to the correlated quark pair density comes from two diagrams of Φ_4.

Quark pair density in the projectile wave function (3)

The leading N_c contribution to the correlated quark pair density in the projectile wave function is given by

$$\left[\frac{dN^P(p,q;\eta_1,\eta_2)}{d^2p\,d^2q\,d\eta_1\,d\eta_2}\right]_{\rm correlated} = -\int_{k\bar{k}l\bar{l}} \left\langle \rho^a(k)\rho^c(\bar{k})\rho^b(l)\rho^d(\bar{l})\right\rangle \Phi_4(k,l,\bar{k},\bar{l};p,q)\ \mathrm{tr}\{\tau^a\tau^b\tau^c\tau^d\}$$

- Adopt MV model for the averaging over the color charges:

$$\left\langle \rho^a(k)\rho^b(p)\right\rangle = (2\pi)^2\mu^2(k)\ \delta^{ab}\ \delta^{(2)}(k+p)$$

- For $k^2 > Q_s^2$: $\mu^2(k)\to\mu^2$ For $k\to 0$: $\mu^2(k)\to 0$.
- estimate the results in the following kinematics:
  (i) rapidity difference between the quarks is relatively large: $\eta_1-\eta_2 \gg 1$
  (ii) the two transverse momenta to be of the same order and much larger than saturation momentum: $|p|\sim|q|\gg Q_s$
- with this estimate answer the basic questions:
  (i) what is the sign of the correlation?
  (ii) how far in rapidity difference does it extend?

Quark pair density and Pauli blocking

The final result:

$$\left[\frac{dN^P(p,q;\eta_1,\eta_2)}{d^2pd^2qd\eta_1 d\eta_2}\right]_{\rm correlated} \simeq -Se^{\eta_2-\eta_1}(\eta_1-\eta_2)^2\frac{\mu^4}{p^4q^4}\frac{g^8N_c^3}{4}\Bigg\{\frac{25\pi^2}{2}q^4\left[\eta_1-\eta_2+\ln\frac{p^2}{Q_s^2}\right]^2\delta^{(2)}(q-p)$$
$$+\pi\left[3\frac{(p^2+q^2)\left[5p^2q^2-3(p\cdot q)^2-(p^2+q^2)p\cdot q\right]}{(q-p)^4}\ln\frac{(p-q)^2}{Q_s^2}+4(\eta_1-\eta_2)p\cdot q\right]\Bigg\}$$

- the correlated contribution is negative! confirms our expectation based on the physics of the Pauli blocking
- the correlation is formally short range in rapidity since it decreases exponentially as a function of the rapidity difference.
- Note that the rate of the decrease in rapidity is tampered by the fourth power of $\eta_1-\eta_2$, so that in practical terms the correlation may extend fairly far in rapidity.

Particle production and Pauli blocking

We have already the expression for the quark pair production cross section:

$$\frac{d\sigma}{d\eta_1\, d^2p\, d\eta_2\, d^2q} = \frac{g^8}{(2\pi)^6}\int -\frac{1}{2}\,\langle\rho^a(x)\rho^b(\bar{x})\rho^c(y)\rho^d(\bar{y})\rangle\Phi_4(x,y,\bar{x},\bar{y};z_1,z_2,\bar{z}_1,\bar{z}_2;\bar{z},\bar{w};p,q)$$

$$\times\,\mathrm{tr}\Big\{[\tau^a - S_A^{a\bar{a}}(x)S_F(z_1)\tau^{\bar{a}}S_F^\dagger(\bar{z})][\tau^c - S_A^{c\bar{c}}(y)S_F(\bar{w})\tau^{\bar{c}}S_F^\dagger(z_2)]$$

$$\times\,[\tau^b - S_A^{b\bar{b}}(\bar{x})S_F(\bar{z}_1)\tau^{\bar{b}}S_F^\dagger(\bar{w})][\tau^d - S_A^{d\bar{d}}(\bar{y})S_F(\bar{z})\tau^{\bar{d}}S_F^\dagger(\bar{z}_2)]\Big\}$$

- The same N_c counting holds for the production cross section. No $\Phi_2\Phi_2$!
- $S(x) = \exp\{igt^a\alpha^a(x)\} \to$ Expand each S in α!
- $\alpha^a(x) = \frac{1}{\nabla^2}(x,y)\rho_T^a(y)$
- Use MV model for $\rho_T\rho_T$ correlator:

$$\langle\rho_T^a(k)\rho_T^b(p)\rangle = (2\pi)^2\lambda^2(k)\delta^{ab}\delta^{(2)}(k+p)$$

- contract both the target and the projectile color charge densities!

Estimates for particle production

Kinematics:

- rapidity difference between the quarks is relatively large: $\eta_1 - \eta_2 \gg 1$
- the two transverse momenta to be of the same order and much larger than saturation momentum: $|p| \sim |q| \sim |p - q| \gg Q_s$
- saturation momentum of the target is smaller than than that of projectile: $Q_T < Q_s$.

$\Downarrow$

This is the regime where correlations existing in the projectile wave function are not strongly distorted by the momentum transfer from the target.

$$\left[\frac{d\sigma}{d^2p\,d^2q\,d\eta_1 d\eta_2}\right]_{\text{correlated}} \simeq -Sg^{12}N_c^5 \frac{\mu^4\lambda^4}{Q_s^2 Q_T^2} e^{\eta_2-\eta_1}(\eta_1-\eta_2)^2 \ln\left(\frac{Q_T^2}{\Lambda^2}\right)\frac{\pi^3}{p^4}$$
$$\times\left\{\frac{50\pi}{16}\ln\left(\frac{Q_s^4}{Q_T^2\Lambda^2}\right)\delta^{(2)}(q-p) + \frac{9Q_s^2}{q^4}\left[\frac{2(p^2+q^2)^2+p^2q^2}{(p-q)^4}\right]\ln\left[\frac{(p-q)^2}{Q_s^2}\right]\right.$$
$$\left. + \frac{9Q_s^2}{2q^4}\left[\ln\left(\frac{q^2}{Q_s^2}\right)+\ln\left(\frac{p^2}{Q_s^2}\right)\right]\right\}$$

Summary

- We have shown that the underlying physics in the "glasma graph calculation" that leads to final state correlations is Bose enhancement of the gluons in the projectile wave function.
- Another physical effect present in the glasma graph calculation is HBT correlations between the gluons far separated in rapidity.
- We have also calculated quark-quark correlated production in the CGC approach. We find that there is a depletion of pair production at like transverse momenta due to Pauli blocking effect.
- Bose enhancement for gluons is long range in rapidity where as Pauli blocking for quarks is short range in rapidity.
- The exponential decay with rapidity difference is tempered by a factor quadratic in rapidity difference, resulting in a dip at $\Delta\eta \sim 2$.
- Quark-quark correlated production turns out to be parametrically $O(\alpha_s^2 N_c)$ realtive to gluon-gluon correlations, which for realistic values of $\alpha_s \sim 0.2$ and $N_c = 3$ results in a mild suppression factor.

BACK UP SLIDES

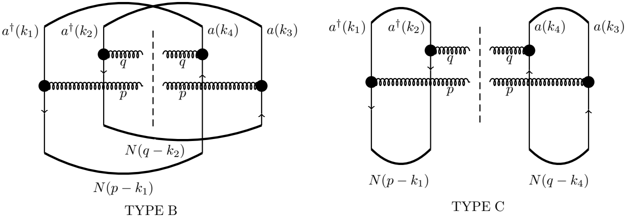
Another physical effect present in the glasma graph calculation:

Hanbury-Brown-Twiss (HBT) correlations between gluons far separated in rapidity!

$\Downarrow$

- leads to a potentially observable effect in the final state mesons which may allow a direct measurement of gluonic size of the proton.
- It correlates gluons with same and opposite transverse momenta (in analogy with the double ridge structure).

The diagrams that will lead to HBT correlations:



For translationally invariant averaging:

- TYPE B $\propto \delta^{(2)}(p-q)$ and TYPE C $\propto \delta^{(2)}(p+q)$

Relaxing the translational invariance $\Rightarrow$ the δ-functions are smeared : $|p \pm q| \simeq R^{-1}$

The standard HBT

The object of interest: **"Two particle correlation function"**

$$C(p,q) = \frac{\frac{dN}{dpdq}}{\frac{dN}{dp}\frac{dN}{dq}}$$

- consider *emission of pions by a chaotic superposition of classical sources*

$$\tilde{J}(\mathbf{p}) = \sum_{i=1}^{N} e^{i\phi_i} e^{ip\cdot x_i} \tilde{J}_0(p - p_i)$$

- each individual classical source is boosted by momentum p_i
- each individual classical source has a random phase ϕ_i

The emission function:

$$\boxed{S(x,K) = \int \frac{d^4y}{2(2\pi)^3} e^{-iK\cdot y} \left\langle J^{*a}\left(x + \frac{1}{2}y\right) J^a\left(x - \frac{1}{2}y\right)\right\rangle}$$

where K is the total momenta of the emitted particles.

$C(p,q)$ can be written in terms of the $S(x,K)$.

Standard HBT vs gluon HBT

Standard HBT

$$\tilde{J}(\mathbf{p}) = \sum_{i=1}^{N} e^{i\phi_i} e^{ip\cdot x_i} \tilde{J}_0(p - p_i)$$

Gluon HBT

$$\tilde{J}^a(\mathbf{p}) = \sum_{i=1}^{N} \left(U_A^i\right)_{ab} e^{ip\cdot x_i} \tilde{J}_0^b(\mathbf{p})$$

- For gluon HBT: $\tilde{J}_0^{*b}(\mathbf{p}) = \tilde{J}_0^b(-\mathbf{p})$ since $J_0^b(x)$ is real!
- For standard HBT: $\mathbf{p}$ is a three vector!
- For gluon HBT: p is transverse only! [The time and longitudinal coordinate dependence of all sources is identical $\Rightarrow \delta(x^+)\delta(x^-)$]

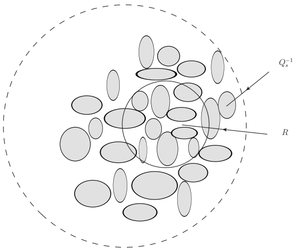
The emission function:

$$\boxed{S(x,K) = \int \frac{d^2y}{2(2\pi)^3} e^{-iK\cdot y} \left\langle J^a\left(x + \frac{1}{2}y\right) J^a\left(x - \frac{1}{2}y\right)\right\rangle}$$

For a Gaussian distribution of the sources, the correlator reads:

$$\boxed{C(q,K) = 1 + \frac{1}{N_c^2 - 1}\left[e^{-R^2q^2} + e^{-R^2K^2}\right]}$$

Transverse structure



- The projectile proton has a distribution of gluon fields with spatial size R (the gluonic size of the proton).
- eikonal scattering with the target $\rightarrow$ gluon fields color rotate : $b_i^a(x) = U^{ab}(x) b_i^b(x)$

For a nucleus with a large saturation scale $Q_s \gg R^{-1}$:

- the eikonal scattering matrix $U(x)$ varies on the spatial scale Q_s^{-1}.
- $\Rightarrow$ the source is color correlated only on scales of order Q_s^{-1}
- color is decorrelated outside due to scattering.

The spatial structure of the source: a collection of independent color sources.

Each source have a transverse size Q_s^{-1}.

The total transverse area of R^2.

The number of independent sources is $N \sim Q_s^2 R^2$.

Properties of the gluon HBT

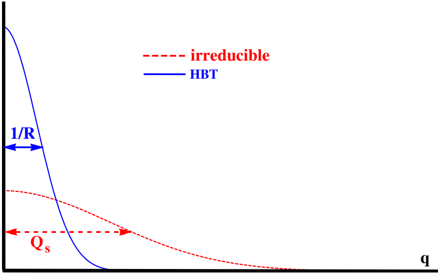
- The correlation is long range in rapidity - it is equally strong when the rapidities of the two gluons are equal or when the difference between the two rapidities is large.
- The correlation is symmetric under reversal of the direction of the transverse momentum of one of the gluons. Thus ,it is strongest when the transverse momenta of the two gluons are either parallel or antiparallel.
- The HBT radius is of the order of the inverse gluonic size of the proton R^{-1}.
- The HBT signal dominates the correlation function (at small momentum difference) when the number of incoherent emitters is large, $N_S = Q_s^2 R^2 \ll 1$.

HBT or Bose Enhancement?

The "glasma graph" calculation contains both type of correlations:

$$\boxed{C(p,q) = \frac{\frac{dN}{dpdq}}{\frac{dN}{dp}\frac{dN}{dq}} = 1 + C_{BE}(p,q) + C_{HBT}(p,q)}$$

- Both $C_{BE}(p,q)$ and $C_{HBT}(p,q)$ are rapidity independent!!
- C_{BE} (the coherent part) contribution is suppressed $\sim 1/R^2Q_T^2$, BUT is "wide" in momentum space $\sim e^{-(p-q)^2/Q_s^2}$ (and $\sim e^{-(p+q)^2/Q_s^2}$).
- $C_{HBT}(p,q)$ is unsuppressed when the number of sources is large: $R^2Q_T^2 \gg 1$ BUT gives a narrow peak $\sim e^{-(p-q)^2R^2}$ (and $\sim e^{-(p+q)^2R^2}$).



Explicit expressions - 1

$$\phi_{s_1s_2}(k,p;\alpha) = \frac{1}{k^2\,[\bar\alpha p^2 \,+\, \alpha(k-p)^2]}\left\{4\alpha\bar\alpha k^2 - [\bar\alpha k\cdot p + \alpha k\cdot(k-p)] + 2i\sigma^3 k\times p\right\}$$

$$\begin{aligned}\Delta = g^4\ \mathrm{tr}\ \Big[& \{\tau^a\tau^{a'}[\alpha^{a'}(x)-\alpha^{a'}(\bar z)] - \tau^{a'}\tau^a[\alpha^{a'}(x)-\alpha^{a'}(z_1)]\}\\ \times & \{\tau^c\tau^{c'}[\alpha^{c'}(y)-\alpha^{c'}(z_2)] - \tau^{c'}\tau^c[\alpha^{c'}(y)-\alpha^{c'}(\bar w)]\}\\ \times & \{\tau^b\tau^{b'}[\alpha^{b'}(\bar x)-\alpha^{b'}(\bar w)] - \tau^{b'}\tau^b[\alpha^{b'}(\bar x)-\alpha^{b'}(\bar z_1)]\}\\ \times & \{\tau^d\tau^{d'}[\alpha^{d'}(\bar y)-\alpha^{d'}(\bar z_2)] - \tau^{d'}\tau^d[\alpha^{d'}(\bar y)-\alpha^{d'}(\bar z)]\}\ \Big].\end{aligned}$$

$$\begin{aligned}\Delta_A = \frac{g^4N_c^5}{16} & \Big\{\langle[\alpha(x)-\alpha(\bar z)][\alpha(\bar y)-\alpha(\bar z)] + [\alpha(x)-\alpha(z_1)][\alpha(\bar y)-\alpha(\bar z_2)]\rangle\Big\}\\ \times & \Big\{\langle[\alpha(\bar x)-\alpha(\bar w)][\alpha(y)-\alpha(\bar w)] + [\alpha(\bar x)-\alpha(\bar z_1)][\alpha(y)-\alpha(z_2)]\rangle\Big\}.\end{aligned}$$

$$\begin{aligned}\Delta_B = \frac{g^4N_c^5}{16} & \Big\{\langle[\alpha(x)-\alpha(\bar z)][\alpha(y)-\alpha(\bar w)] + [\alpha(x)-\alpha(z_1)][\alpha(y)-\alpha(z_2)]\rangle\Big\}\\ \times & \Big\{\langle[\alpha(\bar x)-\alpha(\bar w)][\alpha(\bar y)-\alpha(\bar z)] + [\alpha(\bar x)-\alpha(\bar z_1)][\alpha(\bar y)-\alpha(\bar z_2)]\rangle\Big\}.\end{aligned}$$

Explicit expressions - 2

$$\delta H^{\rho\, g} = \int_0^\infty \frac{dk^+}{2\pi} \frac{d^2k}{(2\pi)^2} \frac{g\, k_i}{\sqrt{2}\,|k^+|^{3/2}} \left[a_i^{\dagger a}(k^+,\, k)\ \rho^a(-k) + a_i^a(k^+,\, k)\ \ \rho^a(k)\right]$$

$$\delta H^{\rho\, qq} = \sum_s \int \frac{dk^+ d^2k\, dp^+ d^2p}{(2\pi)^6} \frac{g^2}{(k^+)^2} \left[d^\dagger_{\alpha\, s}(p^+,\, p)\ \tau^a_{\alpha\beta}\ \bar{d}^\dagger_{\beta\, s}(k^+ \ -\ p^+,\, k - p)\ \rho^a(-k) + h.c.\right]$$

$$\begin{aligned} \delta H^{g\, qq} \quad &= \quad g\, \tau^a_{\alpha\beta} \sum_{s_1, s_2} \int \frac{dp^+\, d^2p\, dk^+\, d^2k}{2^{3/2}\, (2\pi)^6\, (k^+)^{1/2}}\, \theta(k^+ \ -\ p^+)\, \Gamma^i_{s_1\, s_2}(k^+, k, p^+, p) \\ &\times \quad \left[a_i^a(k^+, k)\, d^\dagger_{\alpha,\, s_1}(p^+, p)\, \bar{d}^\dagger_{\beta, s_2}(k^+ - p^+, k - p) + h.c.\right] \end{aligned}$$

$$\Gamma^i_{s_1 s_2}(k^+, k, p^+, p) = \delta_{s_1 s_2}\left[2\frac{k_i}{k^+} - \left(\frac{p_i}{p^+} + \frac{k_i - p_i}{k^+ - p^+}\right) + 2is_1\epsilon^{im}\left(\frac{p_m}{p^+} - \frac{k_m - p_m}{k^+ - p^+}\right)\right]$$

Explicit expressions - 3

$$\langle g|\delta H^{\rho\, g}|0\rangle = \frac{\langle 0|a_i^a(k^+,k)\ \delta H^{\rho g}|0\rangle}{(2\pi)^{3/2}} = \frac{gk_i}{4\pi^{3/2}|k^+|^{3/2}}\rho^a(-k)$$

$$\begin{aligned}\langle q\bar{q}|\delta H^{\rho qq}|0\rangle &= \frac{\langle 0|d_{\alpha s_1}(q^+,q)\ \bar{d}_{\beta s_2}(p^+,p)\delta H^{\rho\, qq}|0\rangle}{(2\pi)^3} \\ &= \frac{g^2\tau^a_{\alpha\beta}}{(2\pi)^3(p^+ + q^+)^2}\rho^a(-p-q)\delta_{s_1 s_2}\end{aligned}$$

$$\begin{aligned}\langle q\bar{q}|\delta H^{g\, qq}|g\rangle &= \frac{\langle 0|d_{\alpha s_1}(p^+,p)\bar{d}_{\beta s_2}(q^+,q)\delta H^q a_i^{a\dagger}(k^+,k)|0\rangle}{(2\pi)^{9/2}} \\ &= g\tau^a_{\alpha\beta}\frac{\Gamma^i_{s_1 s_2}(k^+,k,p^+,p)}{8\pi^{3/2}(k^+)^{1/2}}\delta^{(2)}(p+q-k)\delta(p^+ + q^+ - k^+)\end{aligned}$$

Particle production and Pauli blocking

After all possible contractions at leading N_c, there are two types of contributions to the production cross section:

$$A = -\delta^{(2)}(0)\delta^{(2)}(p-q)\frac{g^{12}N_c^5}{16}\int_0^1 \frac{d\alpha\, d\beta}{(\beta+\bar\beta e^{\eta_1-\eta_2})(\alpha+\bar\alpha e^{\eta_2-\eta_1})}\int_{k,\bar k,l,\bar l}\frac{\mu^2(k)\mu^2(\bar k)\lambda^2(l)\lambda^2(\bar l)}{l^4\bar l^4}$$
$$\times tr\Big\{\Big[\bar\Psi(k,l,p;\alpha)\bar\Psi^*(k,l,p;\alpha)+\Psi(k,l,p;\alpha)\Psi^*(k,l,p;\alpha)\Big]$$
$$\times\Big[\bar\Psi(\bar k,\bar l,p;\beta)\bar\Psi^*(\bar k,\bar l,p;\beta)+\Psi(\bar k,\bar l,p;\beta)\Psi^*(\bar k,\bar l,p;\beta)\Big]\Big\}$$

$$B = -\delta^{(2)}(0)\frac{g^{12}N_c^5}{16}\int_0^1 \frac{d\alpha\, d\beta}{(\beta+\bar\beta e^{\eta_1-\eta_2})(\alpha+\bar\alpha e^{\eta_2-\eta_1})}\int_{k,\bar k,l,\bar l}\frac{\mu^2(k)\mu^2(\bar k)\lambda^2(l)\lambda^2(\bar l)}{l^4\bar l^4}$$
$$\times\delta^{(2)}(k+l-p-\bar k-\bar l+q)\,tr\Big\{\Big[\bar\Psi(k,l,p;\alpha)\bar\Psi^*(k,l,p;\beta)+\Psi(k,l,p;\alpha)\Psi^*(k,l,p;\beta)\Big]$$
$$\times\Big[\bar\Psi(\bar k,\bar l,q;\beta)\bar\Psi^*(\bar k,\bar l,q;\alpha)+\Psi(\bar k,\bar l,q;\beta)\Psi^*(\bar k,\bar l,q;\alpha)\Big]\Big\}$$

with

$$\Psi(k,l,p;\alpha)\equiv[\phi(k+l,p;\alpha)-\phi(k,p-l;\alpha)]$$
$$\bar\Psi(k,l,p;\alpha)\equiv[\phi(k+l,p;\alpha)-\phi(k,p;\alpha)]$$