Nuclear matter at large-Nc

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Various faces of QCD

Swierk 8-9/10/2016

Outline

- Introductory considerations on nuclear matter
- The scalar dibaryon (or dimeron)
- Nuclear matter: does it exist for large-Nc?

  Is in this respect Nc = 3 peculiar or not?
- Conclusions

Introductory remarks/1

Nuclear matter emerges from an interplay of repulsions and attractions (pion ad scalar attractions, omega repulsions)

The energy per baryon (for an infinitely large nuclear matter) is about 16 MeV, which is much smaller than Λ_{QCD} ~ 250 MeV.

Is nuclear matter accidental?

Introductory remarks/2

Real nuclei are less bound due to various effects (Coulomb repulsion, finite size…). Averagely 8 MeV per nucleon.

The smallest nucleus is the deuteron.

$$|\text{Deuteron}\rangle = |\text{space:ground-state}\rangle\, |\uparrow\uparrow\rangle\, |np - pn\rangle \qquad J^P = 1^+$$

It has a binding energy of 2.225 MeV, that is quite small.

Introductory remarks/3

- Is nuclear matter accidental?

- This issue is studied along two directions. First, the existence of a quasi-bound proton-neutron object is investigated. It looks very much like the deuteron, but with quantum numbers I = 1 and J =0.

- Then, we examine what happens when one basic parameters of QCD is changed: the number of colors N_c.

- In both cases we see how elusive is nuclear matter.

The dimeron

Wave function of the scalar dibaryon (dimeron)

Recall:

$$|\text{Deuteron}\rangle = |\text{space:ground-state}\rangle\,|\uparrow\uparrow\rangle\,|np - pn\rangle \quad J^P = 1^+ \quad I = 0$$

We now switch spin and isospin and get the isotriplet:

$$|\text{Dimeron-np}\rangle = |\text{space:ground-state}\rangle\,|\uparrow\downarrow - \downarrow\uparrow\rangle\,|np + pn\rangle$$

$$|\text{Dimeron-nn}\rangle = |\text{space:ground-state}\rangle\,|\uparrow\downarrow - \downarrow\uparrow\rangle\,|nn\rangle$$

$$I = 1$$

$$|\text{Dimeron-pp}\rangle = |\text{space:ground-state}\rangle\,|\uparrow\downarrow - \downarrow\uparrow\rangle\,|pp\rangle$$

$$J^P = 0^+$$

Dimeron: where is it?

- It is not listed in the table of chemical elements: the reason is that it does not bind!

- Yet, it is very close to do it. Just a bit of attraction is missing to generate it.

- It does not exist as a nucleus, but it can be considered as a resonance (not listed in the PDG yet). Its existence is hidden in neutron-proton scattering data.

- Discussion follows the recent paper arXiv: 1603.04312
  by W. Deinet, K. Teilab, F.G., D. Rischke,
  Role of the dibaryon and $f_0(500)$ in proton-proton scattering
  (very recently accepted in Phys. Rev. C)

Dimeron-nucleon coupling

$$|\Phi_R\rangle = |\text{space: ground-state}\rangle \, |\uparrow\downarrow - \downarrow\uparrow\rangle \, |np + pn\rangle$$

$$\boxed{\mathcal{L}_R = iG_R[N^T C\gamma^5 \Phi_R t^1 N + \bar{N}\gamma^5 \Phi_R^* t^1 C \bar{N}^T]}$$

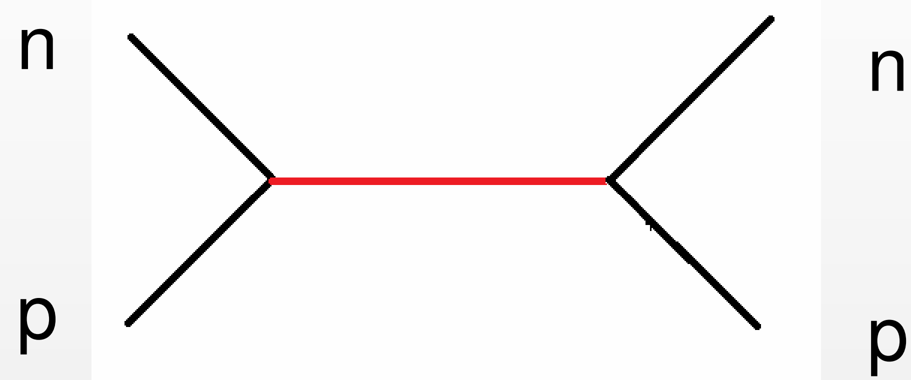
Naïve decay

$$\Gamma(p) = \frac{G_R^2 p}{4\pi}$$

$$p = p(s) = \sqrt{\frac{s^2 + (m_p - m_n)^2 - 2s(m_p^2 + m_n^2)}{4s}}$$

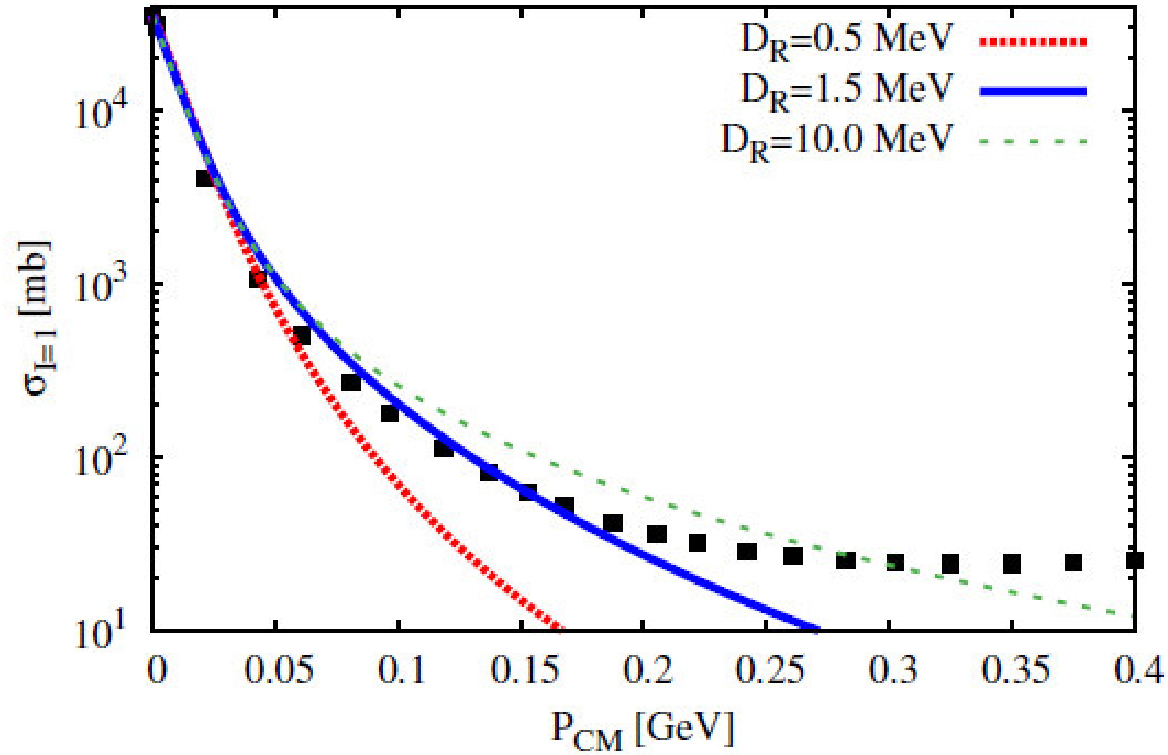
Dibaryon propagator

$$\Delta_R(s) = \frac{i}{s - m_R^2 + i m_R \cdot \Gamma(p)}$$



where m_p and m_n are the proton and the neutron masses, respectively. For a correct description of data it is essential to consider the decay width as a function of p, i.e. $\Gamma(p)$. Setting the decay width to a constant (Breit-Wigner limit, in which $p_R = p(m_R^2)$ is used) is definitely not a good approximation in the present context. Hence, the quantity m_R should not be regarded as a conventional r corresponding to the zero of the real part of the denominator of the propagator, see also the discussion in Sec. III.A.

Dimeron and total cross-section



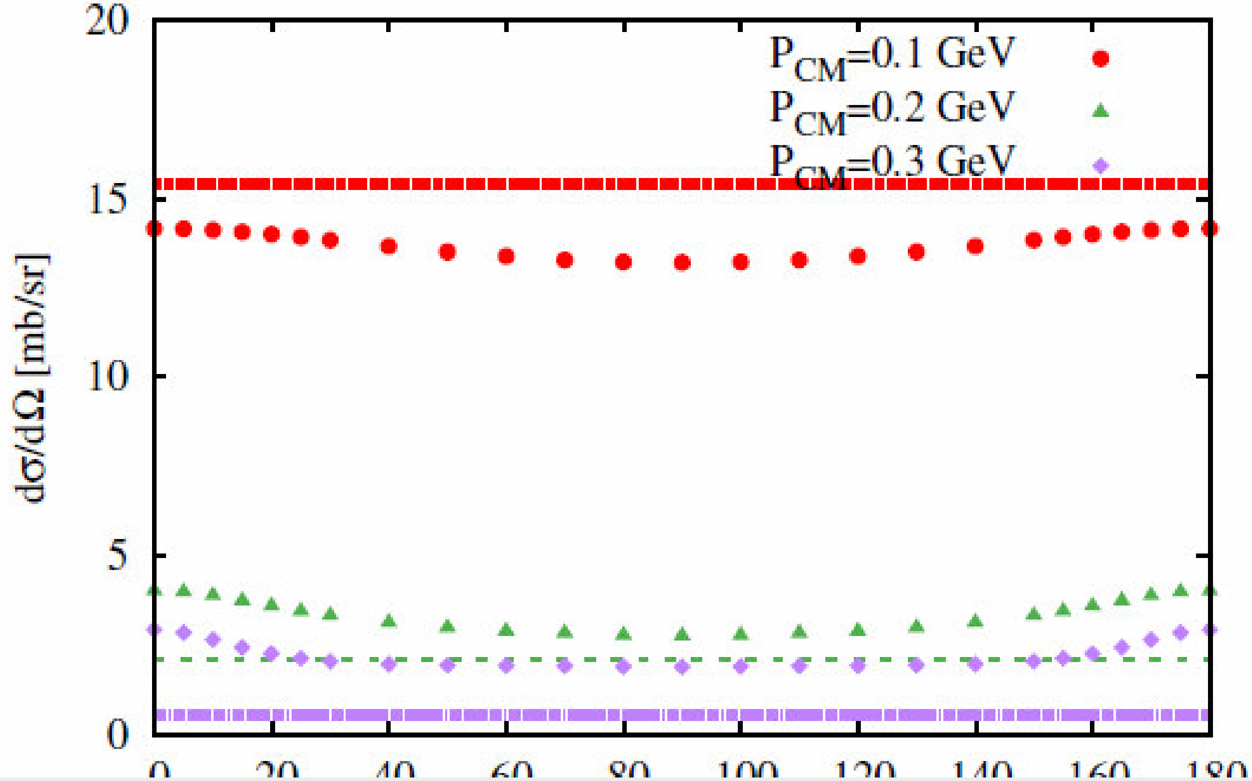
1603.04312

$$D_R = m_R - m_p - m_n$$

The corresponding values for G_R are 1.23, 2.13, 5.5 respectively.

Important: a normal meson exchange delivers cross sections which are order of magnitudes too small!

Dibaryon and differential cross-section



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Spectral function and pole/1

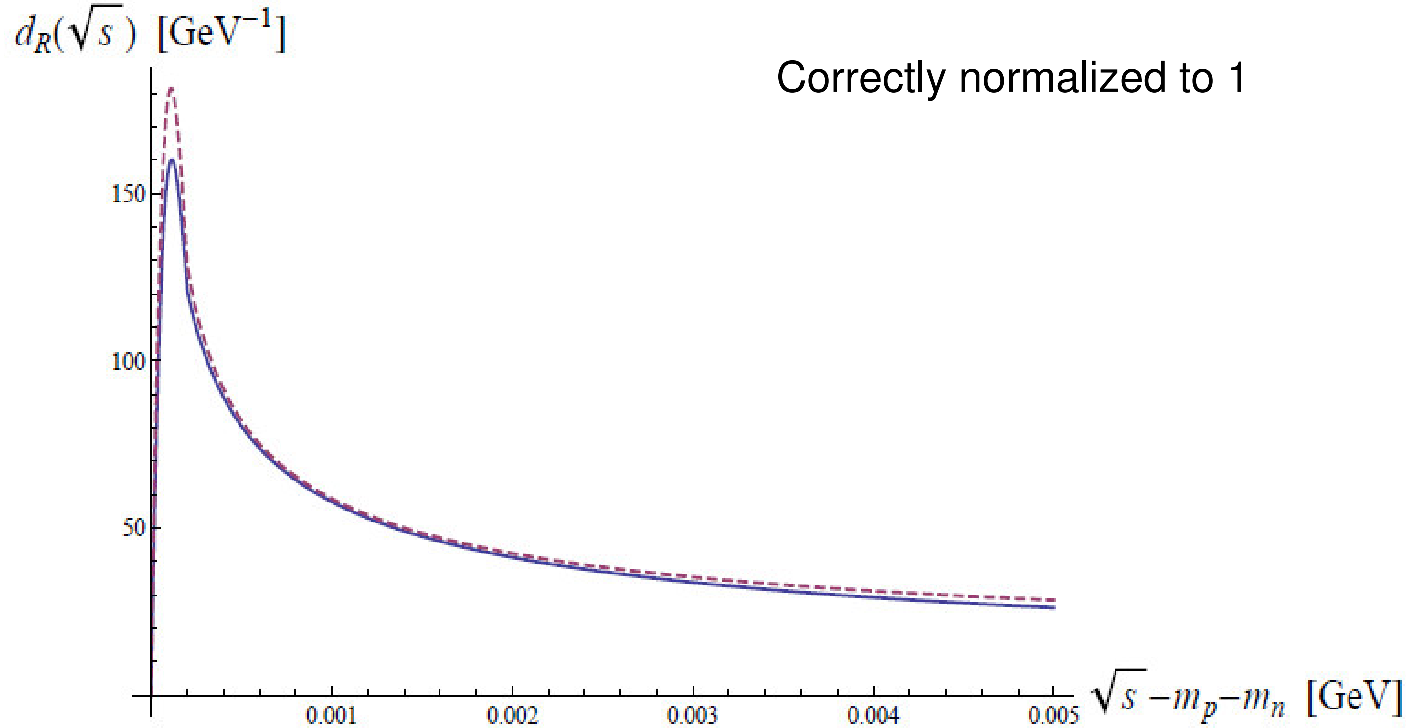
$$\Delta_R^{\text{dressed}}(s) = \frac{1}{s - m_R^2 + \operatorname{Re}\Sigma(s) - \operatorname{Re}\Sigma(m_R^2) + i\operatorname{Im}\Sigma(s)} \, ,$$

$$\operatorname{Re}\Sigma(s) = -PP\int_{m_p+m_n}^{\infty} \frac{1}{\pi}\frac{\operatorname{Im}\Sigma(s')}{s-s'}ds'$$

$$\operatorname{Im}\Sigma(s) = \sqrt{s}\,\Gamma_{\Lambda_R}(p) \qquad \Gamma_{\Lambda_R}(p) = \Gamma(p)e^{-2p^2/\Lambda_R^2}$$

$$d_R(\sqrt{s}) = \frac{2\sqrt{s}}{\pi}\operatorname{Im}\Delta_R^{\text{dressed}}(s)$$

Spectral function and pole/2



Correctly normalized to 1

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$$\sqrt{s_{\rm pole}} = (m_p + m_n + 14.0\ {\rm MeV}) \ \ - i\,(77.4\ {\rm MeV})$$ $\Lambda_R = 0.5$ GeV

$$\sqrt{s_{\rm pole}} = (m_p + m_n + 27.3\ {\rm MeV}) \ \ - i\,(30.9\ {\rm MeV})$$ $\Lambda_R = 0.3$ GeV

Existence and pole position of f0(500)

Complicated PDG history. Existence through the position of the pole.
Now: established.

Citation: K.A. Olive *et al.* (Particle Data Group), Chin. Phys. C, **38**, 090001 (2014) and 2015 update

$f_0(500)$ or σ [g]
was $f_0(600)$

$I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = (400\text{–}550)$ MeV
Full width $\Gamma = (400\text{–}700)$ MeV

f_0(500) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$\pi\pi$	dominant	–
$\gamma\gamma$	seen	–

Citation: K.A. Olive *et al.* (Particle Data Group), Chin. Phys. C, **38**, 090001 (2014) and 2015 update

$f_0(500)$ or σ
was $f_0(600)$

$I^G(J^{PC}) = 0^+(0^{++})$

A REVIEW GOES HERE – Check our WWW List of Reviews

f_0(500) T-MATRIX POLE $\sqrt{s}$

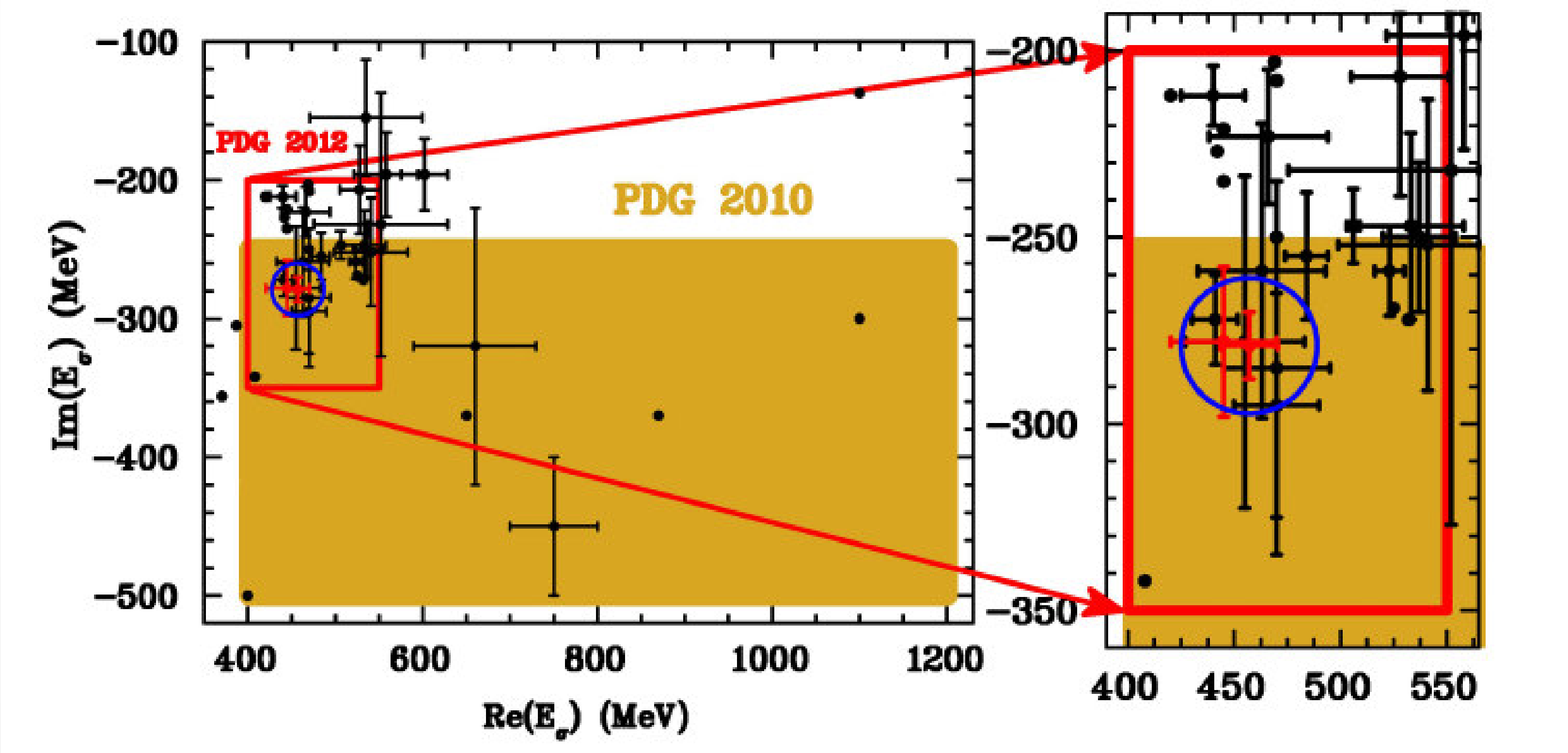
Note that $\Gamma \approx 2\,\mathrm{Im}(\sqrt{s_{\mathrm{pole}}})$.

VALUE (MeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
(400–550)–i(200–350) OUR ESTIMATE			

$$\sqrt{s_{pole}} = M - i\frac{\Gamma}{2}$$

Existence and pole position of f0(500)

From 2010 to 2012: update



See the review of J.R. Pelaez (Madrid U.), e-Print: **arXiv:1510.00653**
A review on the status of the non-ordinary f0(500) **resonance**

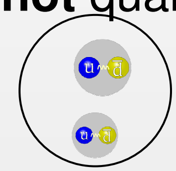
The light scalar mesons: what are they?

$$a_0(980) \quad k(800) \quad f_0(980) \quad \boxed{f_0(500)}$$

$$J^{PC} = 0^{++}$$

Various studies show that these states are **not** quark-antiquark states.

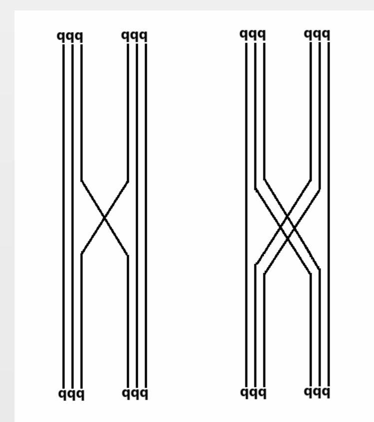
They can be meson-meson molecules



and/or diquark-antidiquark states.

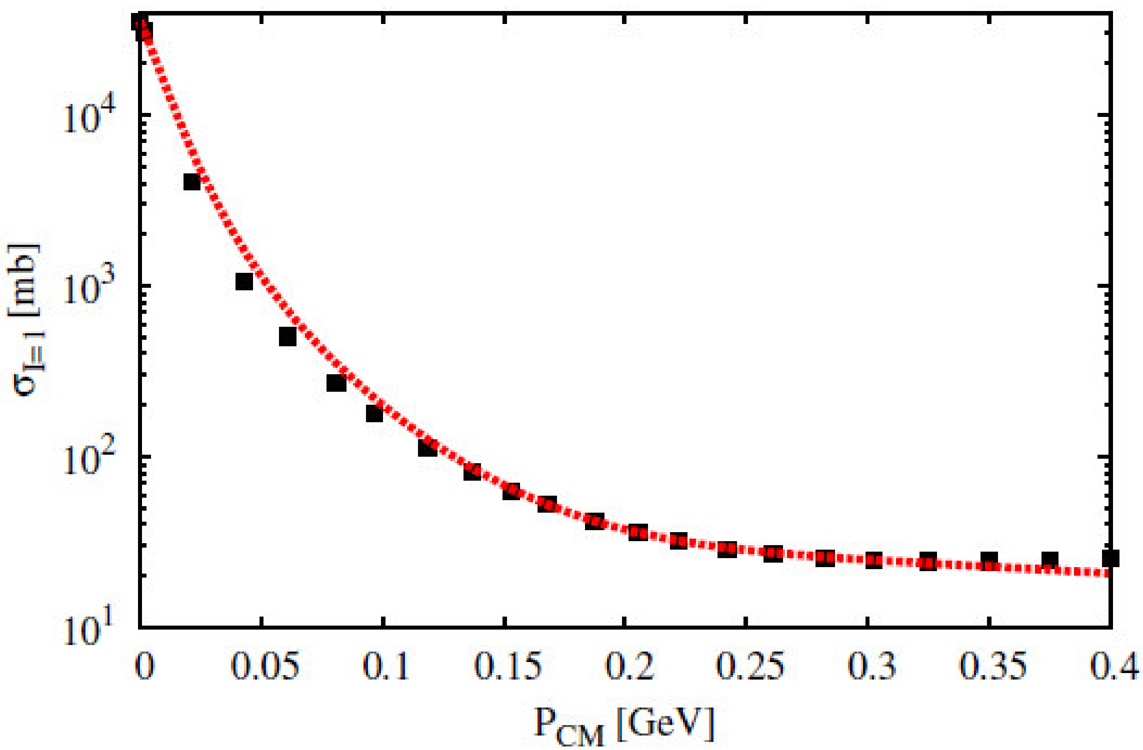
In both cases we have **four-quark** objects.

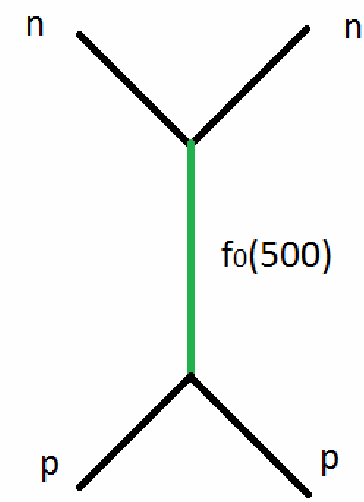
f0(500) is the lighest scalar states: important in nuclear interaction and in studies of chiral symmetry restorations.



np-scattering: dibaryon and f0(500)

$$\mathcal{L}_{\chi NN} = -2a\chi NN$$





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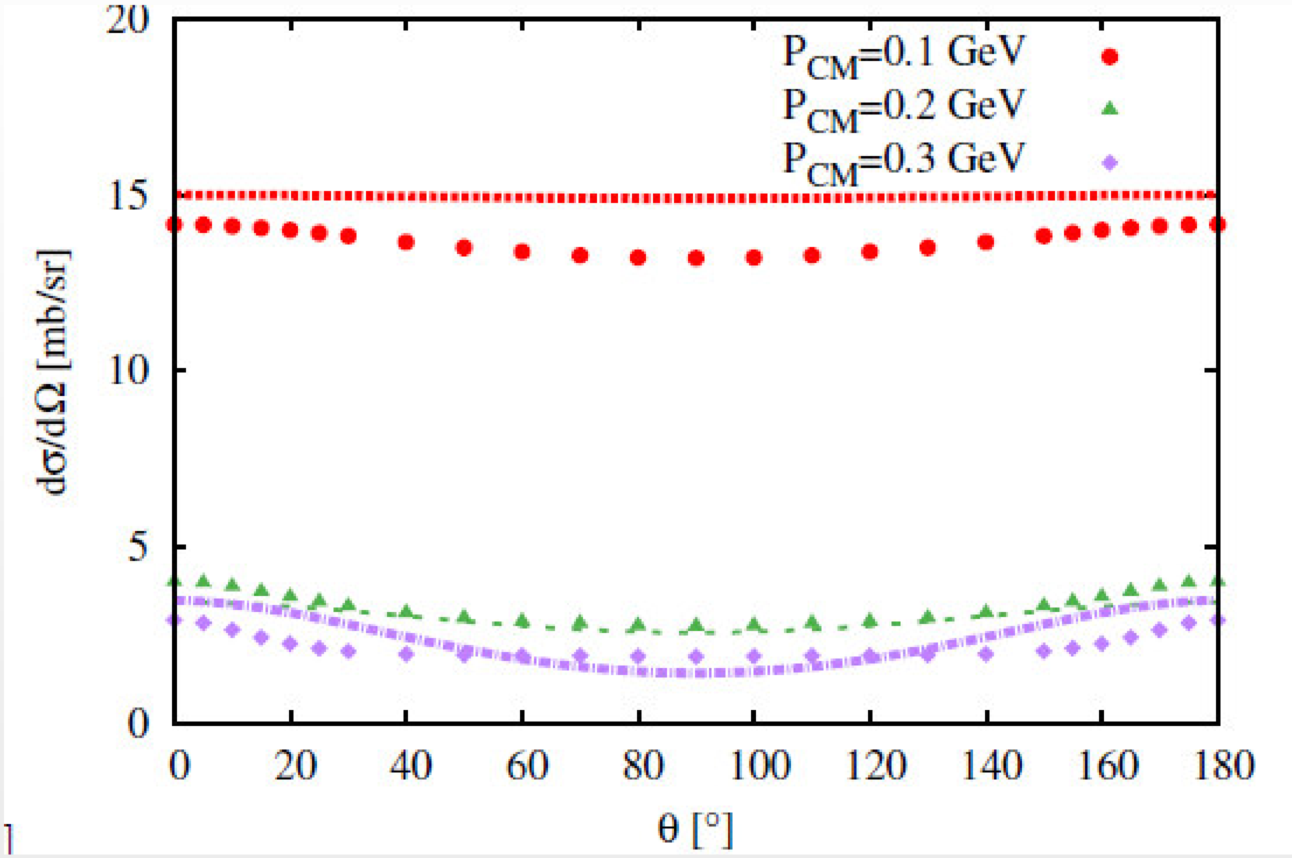
Cross sections for the $I = 1$ np-scattering. The theoretical curve in figure (a) is calculated for scattering via χ exchange in addition to the $^1S_0$ resonance with $D_R = 1.8$ MeV and $G_R = 2.27$. The mass of the χ meson is set to 525 MeV and its coupling g_χ to 487 MeV.

np-scattering: dibaryon and f0(500)



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Chiral model of QCD
extended Linear Sigma Model (eLSM)
Baryon sector

$$\begin{aligned}\mathcal{L}_{eLSM} &= \bar{\Psi}_{1L} i\gamma_\mu D_{1L}^\mu \Psi_{1L} + \bar{\Psi}_{1R} i\gamma_\mu D_{1R}^\mu \Psi_{1R} + \bar{\Psi}_{2L} i\gamma_\mu D_{2R}^\mu \Psi_{2L} + \bar{\Psi}_{2R} i\gamma_\mu D_{2L}^\mu \Psi_{2R} \\ &- \hat{g}_1(\bar{\Psi}_{1L}\Phi\Psi_{1R} + \bar{\Psi}_{1R}\Phi^+\Psi_{1L}) - \hat{g}_2(\bar{\Psi}_{2L}\Phi^+\Psi_{2R} + \bar{\Psi}_{2R}\Phi\Psi_{2L}) \\ &- a\chi(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} + \bar{\Psi}_{2R}\Psi_{1L})\end{aligned}$$

$$D_{1R}^\mu = \partial^\mu - ic_1 R^\mu \; , \; D_{1L}^\mu = \partial^\mu - ic_1 L^\mu$$

$$D_{2R}^\mu = \partial^\mu - ic_2 R^\mu , \; D_{2L}^\mu = \partial^\mu - ic_2 L^\mu$$

S. Gallas, F. G., D. H. Rischke, Phys. Rev. D. 82 (2010) 014004 ; arXiv: 0907.5084
S. Gallas, F. G., G. Pagliara, Nucl. Phys. A 872 (2011), arXiv: 1105.5003

Four-quark state χ=f0(500) coupled in chirally invariant way.
But now also (pseudo)scalar and (axial-)vector mesons are present.

at each baryon-baryon-meson vertex:

$$F(q^2) = \exp\left(-\frac{\sqrt{(q^2 - m_i^2)^2}}{\Lambda_{cut}^2}\right) , \tag{18}$$

where q^2 is the momentum transfer involved in the process (q is the four-momentum of the exchanged meson, and m_i its mass, $i = \pi, \rho, ...,$) The parameter Λ_{cut} is an hadronic energy scale, which is expected to be of the order of $\Lambda_{cut} \sim 1$ GeV .

Results of the eLSM model

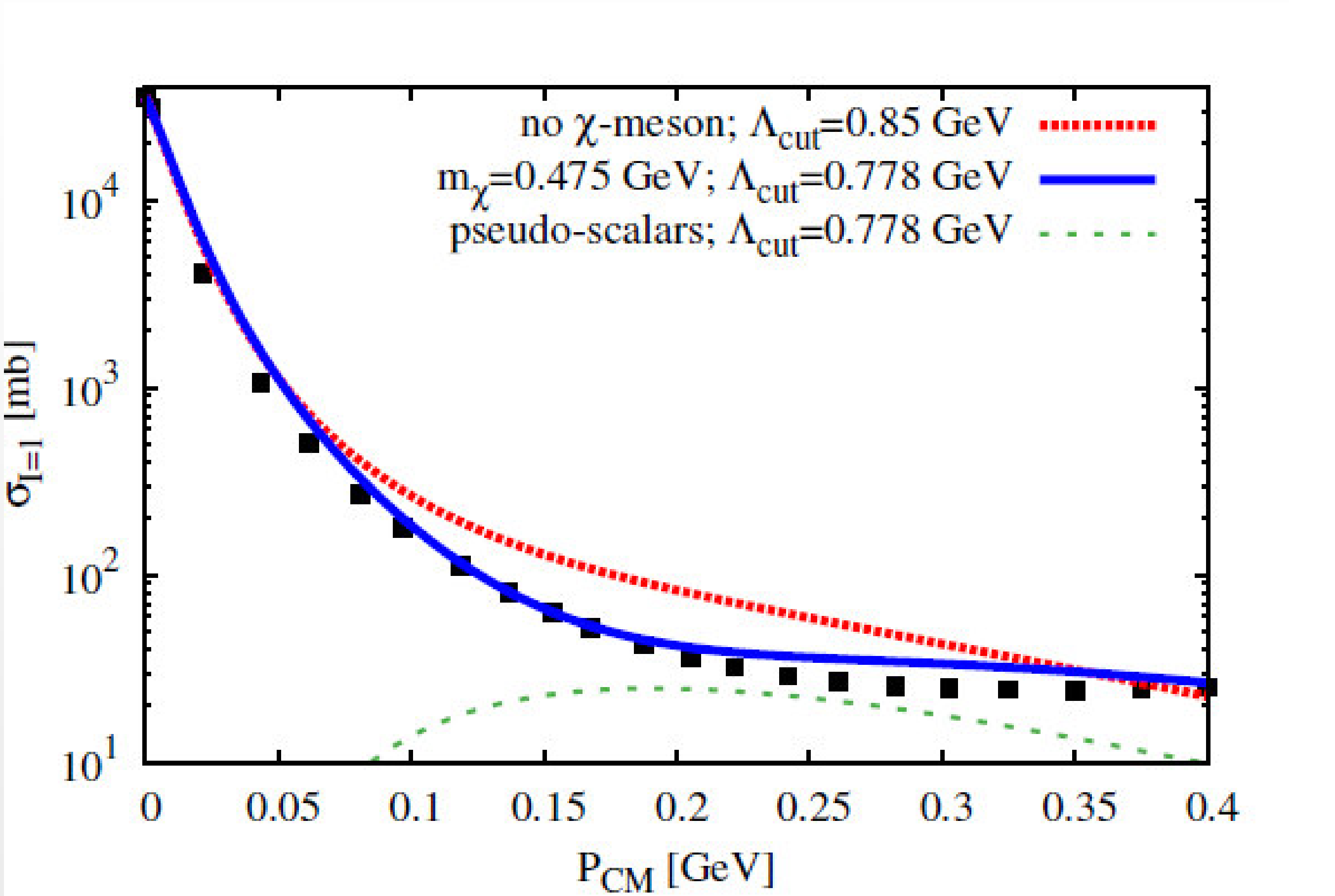

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FIG. 7: Cross sections for the $I = 1$ np-scattering. The theoretical curves in figure (a) are calculated for scattering through $\sigma, a_0, \pi, \eta, \omega, \rho, f_1, a_1$ exchange as well as via the ${}^1S_0$ resonance with D_R=1.6 MeV. The dashed curve shows the case where no χ meson is included (Λ_{cut}=0.85 GeV), while the solid curve shows the case where the χ meson with mass of m_χ=475 MeV and coupling g_χ=6.75 is included (Λ_{cut}=0.778 GeV).The corresponding differential cross sections are shown in figures (b) and (c).

Dibaryon (or dimeron) – conclusive remarks

- There is a pole in the complex plane: this is the important point.

For effective approaches: U. van Kolck, Prog. Part. Nucl. Phys. **43** (1999) 337 [nucl-th/9902015].
P. F. Bedaque and U. van Kolck, Ann. Rev. Nucl. Part. Sci. **52** (2002) 339 [nucl-th/0203055].

- State very close to threshold (just a very bit is missing to be a regular bound state)

- Yet, the effect of this resonance in neutron-proton scattering is visible.

- Hint toward the existence of the nn-dimeron state was also found in experiments.

A. Spyrou *et al.*, Phys. Rev. Lett. **108**, 102501 (2012)

Large-Nc and nuclear matter

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Short recall of large-N_c properties

$$N_c \to \infty, \qquad g_{QCD}^2 \, N_c \to \text{const.}$$

Quark-antiquark meson masses: $M_{q\bar{q}} \propto N_c^0$

n-leg meson Interaction: $A_{n-q\bar{q}} \propto N_c^{-(n-2)/2}$

Baryon mass: $M_B \propto N_c$

Baryon-meson interaction: $A_{q\bar{q}-Baryon} \propto \sqrt{N_c}$

G. 't Hooft, Nucl. Phys. B72, 461 (1974),

E. Witten, Nucl. Phys. B160, 57 (1979).

Nuclear matter: Walecka model

$$L_{Walecka} = \text{free} - \text{part} + g_S S \bar{\psi}\psi + g_{\omega} \omega_{\mu} \bar{\psi} \gamma^{\mu} \psi$$

$$\psi = \text{Nucleon field}$$

$\omega \equiv \omega(782)$ vector repulsion: ω is a quark-antiquark state

$$g_{\omega} \propto \sqrt{N_c}, \quad m_{\omega} \propto N_c^0$$

S = f0(500) scalar attraction: but what is S?

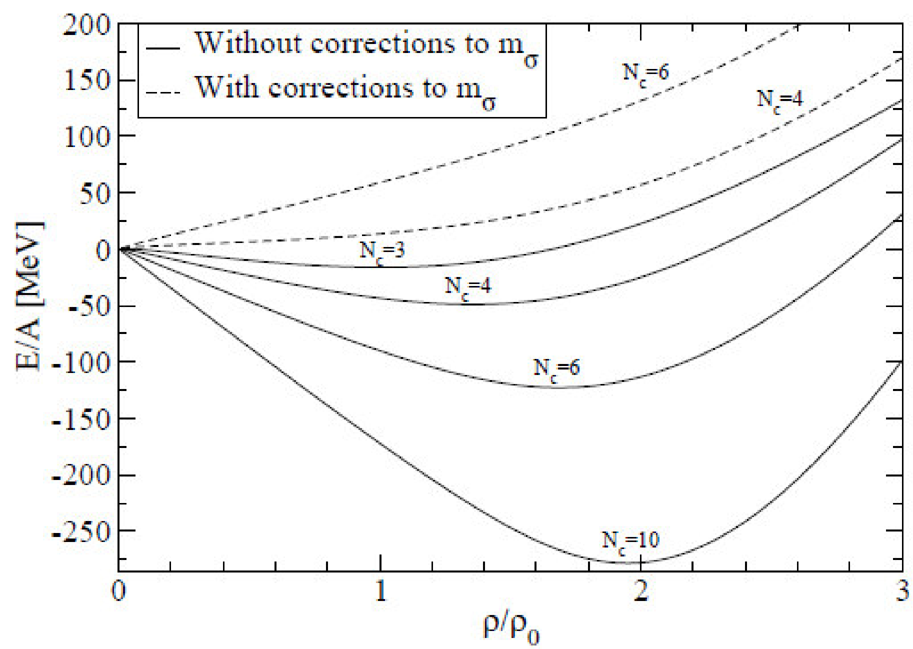
$$g_S \propto ?, \quad m_S \propto ?$$

S is most probably not a quarkonium...but a four-quark object.

This affects the large-Nc properties.

Old-fashioned quarkonium assignment for S = f0(500) (most probably wrong)

$$S \equiv \sigma =, \quad g_\sigma \propto \sqrt{N_c}\,, \quad m_\sigma \propto N_c^0$$

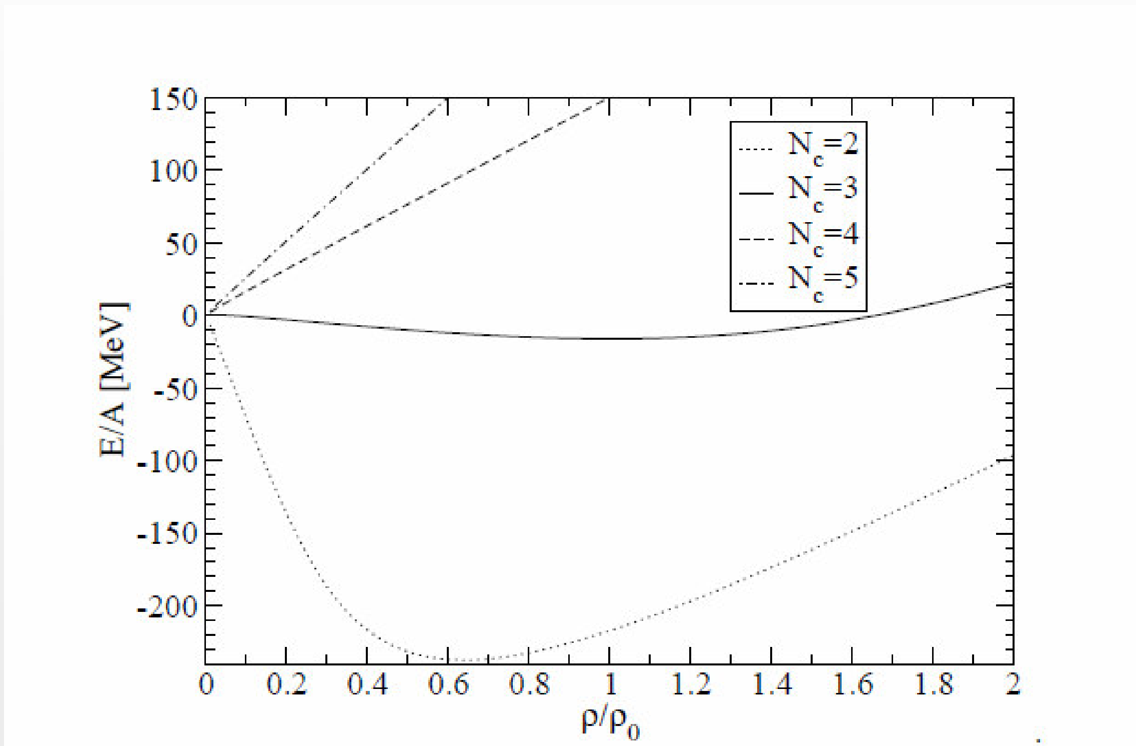


1102.3367 [hep-ph]

Nuclear matter exists for large Nc. Binding energy increases.

Tetraquark interpretation for S = f0(500)

$$S \equiv Tq \equiv [\bar{u},\bar{d}][u,d], \quad g_{Tq} \propto e^{-N_c}, \quad m_{Tq} \propto N_c$$



1102.3367 [hep-ph]

- Nuclear matter does not survive in the löarge-Nc limit.
- Already for Nc=4 : no binding.

Final result in the mean-field approx.

The non-existence of nuclear matter at large-Nc holds also for all the other possible interpretation of the resonance S = $f_0(500)$.

- S as glueball
- S as pion-pion enhancement
- in other mixing scenario…

Nuclear matter (as we know it) is a peculiar property of $N_c = 3$

How, care is needed...

Details in: L. Bonanno and F.G., Nucl. Phys. A 859 49-62 **arXiv:1102.3367** [hep-ph]

Going beyond mean field

- The described result is valid in the mean-field approximation (in which pions do not enter).

- Yet, the pion attraction increases with Nc. Being the pion the lightest state also for large Nc, it turns out that for Nc large enough bound states emerges.

- For instance, preliminary studies with the deuteron show that a different type of nuclear matter exists for Nc larger than (ca) 14. (calculation with Luca Bonanno, unpublished).

$$V \propto -\frac{g_{NN\pi}^2}{r}e^{-m_\pi r} + \frac{g_{NN\omega}^2}{r}e^{-m_\omega r} + \dots$$

$$g_{NN\pi}^2 \propto N_c \ , \ g_{NN\omega}^2 \propto N_c$$

- In the end, we expect that nuclear appears again for Nc large enough.

Conclusions…

- Nuclear matter is very peculiar and “weekly” bound.

- The brother of the deuteron, the dimeron, does not exist as a bound state, but as a resonance which strongly affects neutron-proton scattering data.

  A pole in the complex plane exists.

- Nuclear matter exists for Nc = 3, but would not for Nc between 4 and (ca) 14. Only for very large Nc a strongly bound nuclear matter is (probably) realized.

… and outlook

• Ongoing work (with G. Pagliara, Univ. Ferrara): phase transition to quarks (and gluons) at high density takes place.

• Large-Nc suggest that phase transition takes place very late (hadronic phase, eventually in some sort of chirally restored phase) is valid up to very high density. We find that the phase transition takes place at even larger density than previously thought ($\mu_{crit} \sim N_c^2$)

• An hardening of the eos takes place. Eventually important to understand the large masses of neutron stars.

Thank You!

Two aspects of Large-Nc

- Chiral phase transition at nonzero temperature

It is expected that $T_c \propto \Lambda_{QCD} \propto N_c^0$ (just as $T_{\text{deconfinement}}$)

- Nuclear matter: does it exist for Large-Nc?

Is in this respect Nc = 3 peculiar or not?

Chiral phase transition at nonzero T with the NJL-model

$$\mathscr{L}_{NJL} = \bar{\psi}(\imath\partial\!\!\!/ - m)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}\imath\gamma_5\psi)^2\right]$$

$$T_c \simeq \Lambda\sqrt{\frac{3}{\pi^2}}\sqrt{1-\frac{\pi^2}{2\Lambda^2 G N_c}} \sim \# N_c^0 + \#\frac{1}{N_c}\ldots$$

S. P. Klevansky, Rev. Mod. Phys. 64, 649-708 (1992)

- This is the expected result.
- The same result holds for generalizations to more flavors, and PNJL-type models

Chiral phase transition at nonzero T within a purely hadronic model

$$T_c \propto \sqrt{N_c}$$

- The result is general: no matter how complicated is the hadronic model.
- This result is in contraddiction with the NJL one
- It is based on the assumptions that the parmeters of the model do not depend on temperature

Example: linear σ-Model

$$\mathscr{L}_\sigma = \frac{1}{2}(\partial_\nu \Phi)^2 + \frac{1}{2}\mu^2\Phi^2 - \frac{\lambda}{4}\Phi^4 + \epsilon\sigma, \qquad (\mu^2 > 0,\ \epsilon \to 0^+)$$

$$\Phi = \tau_0\sigma + \imath\tau_i\pi_i$$

implementing large N_c scaling

$$\mu \to \mu, \qquad \lambda \to \left(\frac{3}{N_c}\right)\lambda$$

$$\Rightarrow \mathscr{L}_\sigma(N_c) = \frac{1}{2}(\partial_\nu \Phi)^2 + \frac{1}{2}\mu^2\Phi^2 - \frac{\lambda}{4}\frac{3}{N_c}\Phi^4 + \epsilon\sigma, \qquad (\mu^2 > 0,\ \epsilon \to 0^+)$$

$$T_c \propto \sqrt{N_c}$$

How to cure the problem?

$$\Rightarrow \mathscr{L}_\sigma(N_c) = \frac{1}{2}(\partial_\nu \Phi)^2 + \frac{1}{2}\mu(T)^2\Phi^2 - \frac{\lambda}{4}\frac{3}{N_c}\Phi^4 + \epsilon\sigma, \quad (\mu(0)^2 > 0,\ \epsilon \to 0^+)$$

- μ has dimension [energy]^2, $\mu^2 \to \mu(T)^2$

e.g.:

$$\mu(T)^2 = \mu^2\left(1 - \frac{T^2}{T_d^2}\right), \qquad T_d \sim \Lambda_{QCD} \sim N_c^0$$

this leads to a modified large N_c dependency of T_c

$$\Rightarrow T_c = T_d \frac{1}{\sqrt{1 + 2\lambda \frac{T_d^2}{\mu^2}\frac{3}{N_c}}}, \qquad \lim_{N_c \to \infty} T_c = T_d \sim N_c^0$$

- The simple Ansatz can find a justification with polyakov-loop.
- Generalization to realistic hadronic models is possible (and indeed easy...)

Details will be in: A. Heinz, F.G., D. Riscchke (currently in preparation)

Origin of the nucleon mass

$$m_N = m_N(\phi, \chi_0) = \sqrt{a^2\chi_0^2 + ((\hat{g}_1 + \hat{g}_2)\phi/4)^2} + ((\hat{g}_1 - \hat{g}_2)\phi/4)$$

If $a\chi_0 = 0 \rightarrow m_N = \frac{1}{2}\hat{g}_1\phi$ old linear sigma models, but they do not work

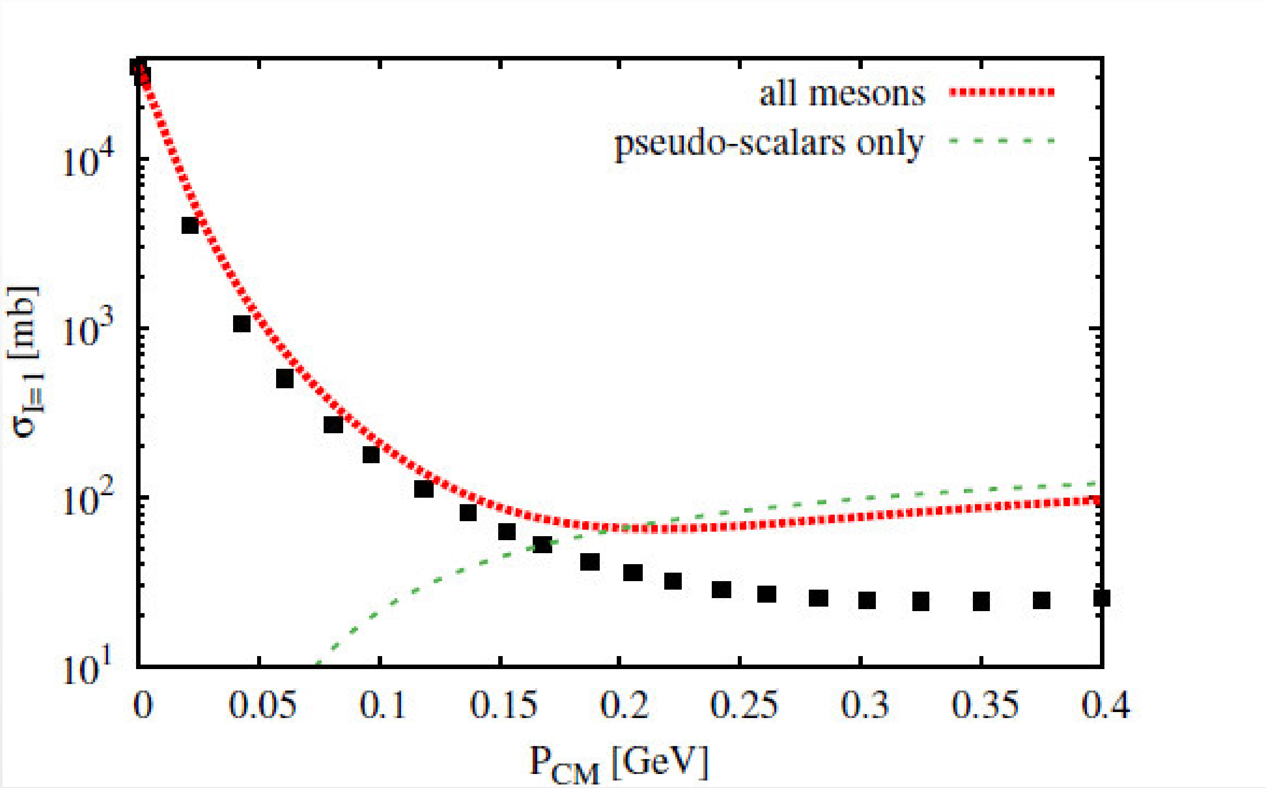
$$m_0 = a\chi_0 \simeq 500 \text{ MeV}$$

is the mass which contribution arising from the four-quark condensate

This is the mechanism generating to 95% of the visible Universe's visible mass.

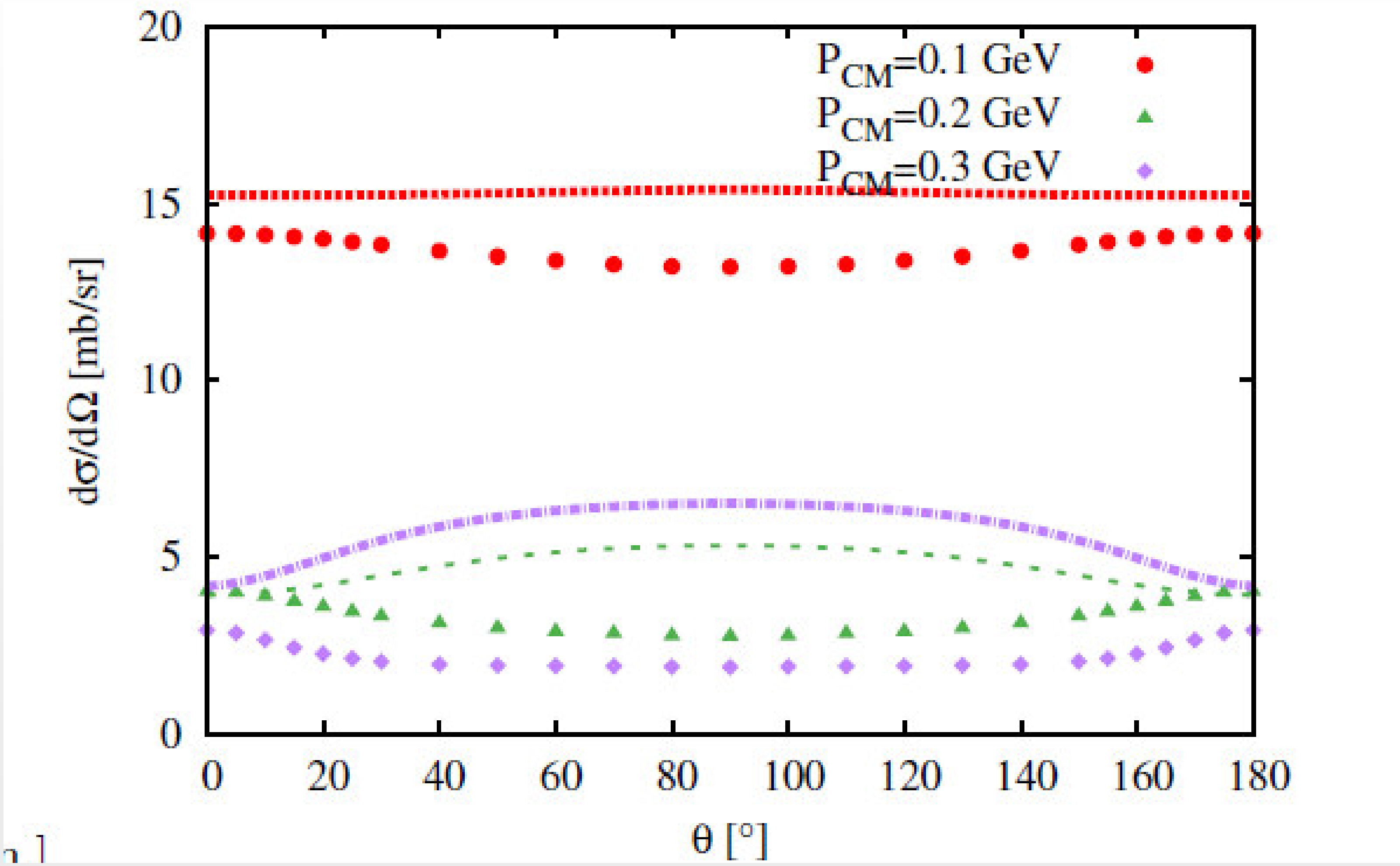
It is not the Higgs!!! Higgs is only responsible for the remaining 5%.

Total cross-section: something is wrong...



Cross sections for the $I = 1$ np-scattering. The dashed curve in (a) is calculated for scattering via exchange of the 9 mesons included in equation (16) (m_χ=0.475 GeV and g_χ=4.87) in addition to the $^1S_0$ resonance with D_R=1.8 MeV and G_R=2.26. The corresponding differential cross sections are shown in (b).

Differential cross-section: something is also wrong...



Cross-section with form factor

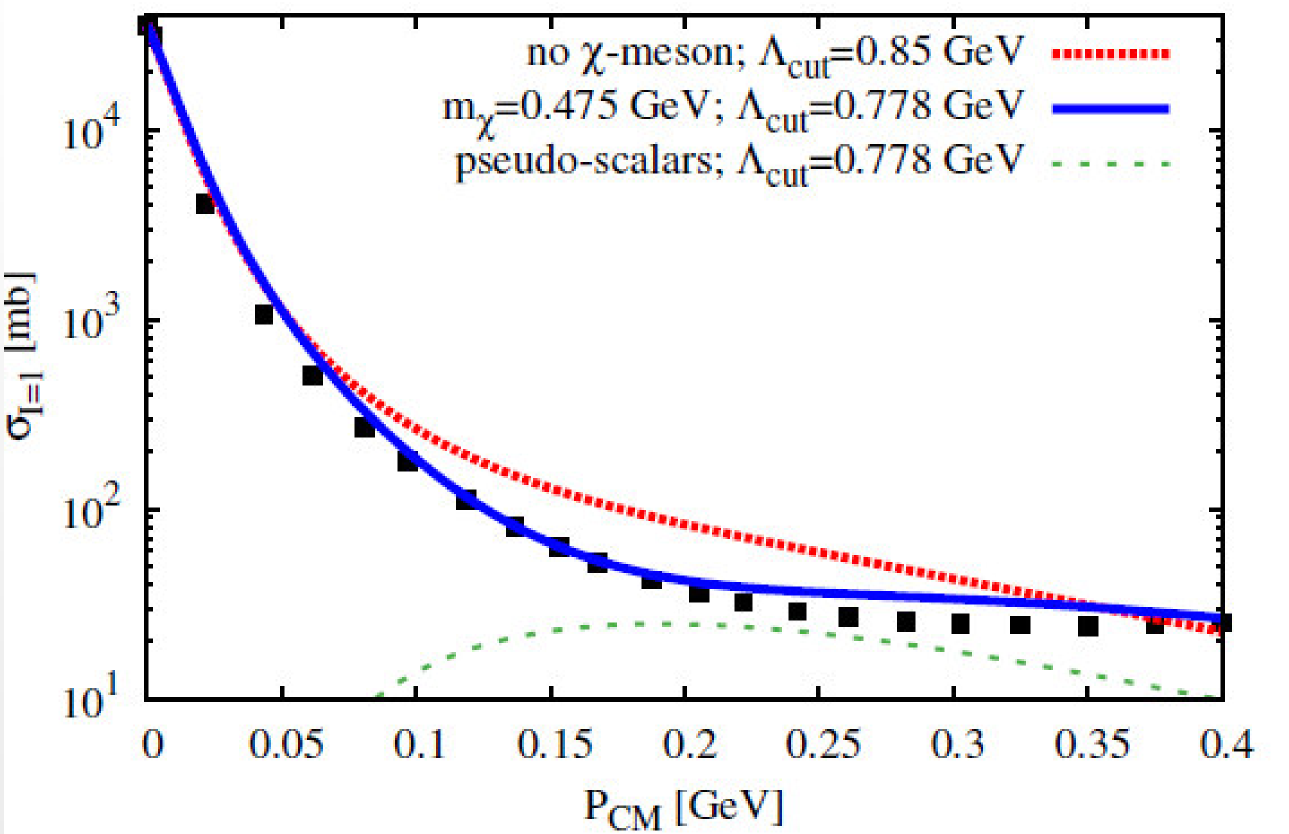
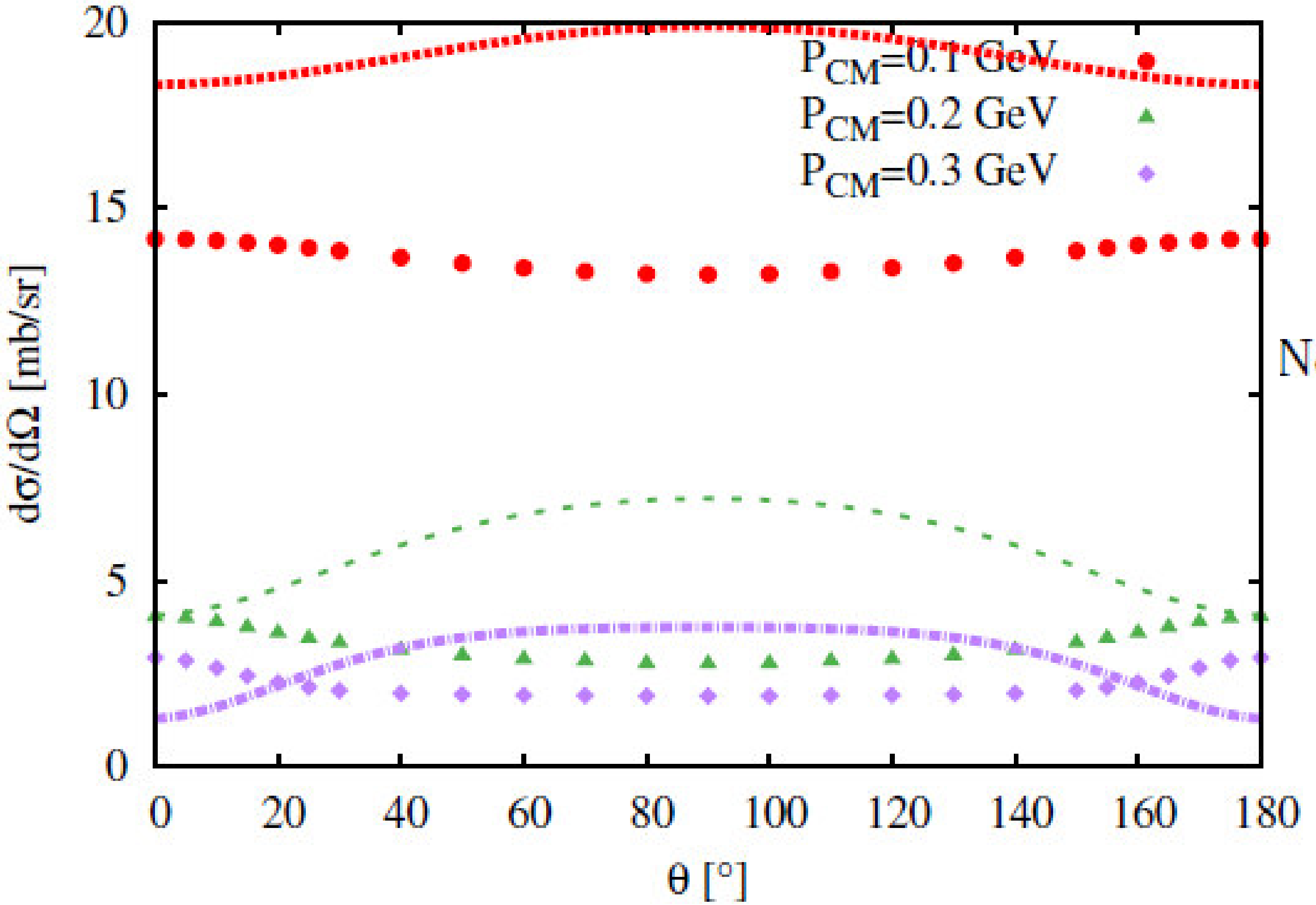


FIG. 7: Cross sections for the $I = 1$ np-scattering. The theoretical curves in figure (a) are calculated for scattering through $\sigma, a_0, \pi, \eta, \omega, \rho, f_1, a_1$ exchange as well as via the $^1S_0$ resonance with D_R=1.6 MeV. The dashed curve shows the case where no χ meson is included (Λ_{cut}=0.85 GeV), while the solid curve shows the case where the χ meson with mass of m_χ=475 MeV and coupling g_χ=6.75 is included (Λ_{cut}=0.778 GeV).The corresponding differential cross sections are shown in figures (b) and (c).

Differential cross-section without f0(500)



No χ−meson

Differential cross-section with f0(500)