Exclusive photoproduction of a $\gamma \rho$ pair with a large invariant mass

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Various faces of QCD 2

Świerk, 8-9 October 2016

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arXiv:1609.03830
Transversity of the nucleon using hard processes

What is transversity?

- Transverse spin content of the proton:
  \[ |\uparrow\rangle(x) \sim |\rightarrow\rangle + |\leftarrow\rangle \]
  \[ |\downarrow\rangle(x) \sim |\rightarrow\rangle - |\leftarrow\rangle \]
  spin along \( x \) helicity states

- Observables which are sensitive to helicity flip thus give access to transversity \( \Delta T q(x) \). Poorly known.

- Transversity GPDs are completely unknown experimentally.

- For massless (anti)particles, chirality = (-)helicity

- Transversity is thus a chiral-odd quantity

- Since (in the massless limit) QCD and QED are chiral-even \( (\gamma^\mu, \gamma^\mu\gamma^5) \), the chiral-odd quantities \( (1, \gamma^5, [\gamma^\mu, \gamma^\nu]) \) which one wants to measure should appear in pairs
Transversity of the nucleon using hard processes: using a two body final state process?

How to get access to transversity GPDs?

- the dominant DA of $\rho_T$ is of twist 2 and chiral-odd ($[\gamma^\mu, \gamma^\nu]$ coupling)
- unfortunately $\gamma^* N^\uparrow \rightarrow \rho_T N' = 0$
  - This cancellation is true at any order: such a process would require a helicity transfer of 2 from a photon.
- lowest order diagrammatic argument:

\[
\gamma^\alpha [\gamma^\mu, \gamma^\nu] \gamma_\alpha \rightarrow 0
\]

[Diehl, Gousset, Pire], [Collins, Diehl]
Transversity of the nucleon using hard processes: using a two body final state process?

Can one circumvent this vanishing?

- This vanishing only occurs at twist 2

- At twist 3 this process does not vanish [Ahmad, Goldstein, Liuti], [Goloskokov, Kroll]

- However processes involving twist 3 DAs may face problems with factorization (end-point singularities)
  can be made safe in the high-energy $k_T$—factorization approach
  [I. Anikin, D. Ivanov, B. Pire, L.Sz., S.Wallon]

- One can also consider a 3-body final state process [D. Ivanov, B. Pire, L.Sz., O. Teryaev], [R. Enberg, B. Pire, L. Sz.], [M. El Beiyad, B. Pire, M. Segond, L.Sz, S. Wallon]
Probing GPDs using ρ meson + photon production

- We consider the process $\gamma N \rightarrow \gamma \rho N'$
- Collinear factorization of the amplitude for $\gamma + N \rightarrow \gamma + \rho + N'$ at large $M_{\gamma \rho}^2$

large angle factorization
à la Brodsky Lepage
Probing chiral-even GPDs using $\rho$ meson + photon production

Processes with 3 body final states can give access to chiral-even GPDs

**Diagram:**

- $T_H$
- $N$ and $N'$
- $x + \xi$ and $x - \xi$
- $t'$
- $M_{\gamma\rho}^2$
- $\rho_L$
- Chiral-even twist 2 DA

**Equation:**

$$t (\text{small})$$

**Text:**

- Access to GPDs through a 3 body final state
- Non-perturbative ingredients
- Computation
- Results
- Conclusion
Probing chiral-odd GPDs using $\rho$ meson + photon production

Processes with 3 body final states can give access to chiral-odd GPDs

chiral-odd twist 2 DA

chiral-odd twist 2 GPD
Probing chiral-odd GPDs using $\rho$ meson + photon production

Processes with 3 body final states can give access to chiral-odd GPDs

How did we manage to circumvent the no-go theorem for $2 \rightarrow 2$ processes?

Typical non-zero diagram for a transverse $\rho$ meson

the $\sigma$ matrices (from DA and GPD sides) do not kill it anymore!
Master formula based on leading twist 2 factorization

\[ A \propto \int_{-1}^{1} dx \int_{0}^{1} dz \ T(x, \xi, z) \times H(x, \xi, t) \Phi_\rho(z) + \cdots \]

- Both the DA and the GPD can be either chiral-even or chiral-odd.
- At twist 2 the longitudinal \( \rho \) DA is chiral-even and the transverse \( \rho \) DA is chiral-odd.
- Hence we will need both chiral-even and chiral-odd non-perturbative building blocks and hard parts.
Kinematics to handle GPD in a 3-body final state process

- Use a Sudakov basis:
  - Light-cone vectors $p$, $n$ with $2p \cdot n = s$
- Assume the following kinematics:
  - $\Delta_{\perp} \ll p_{\perp}$
  - $M^2$, $m_{\rho}^2 \ll M_{\gamma\rho}^2$

- Initial state particle momenta:
  - $q^{\mu} = n^{\mu}$, $p_1^{\mu} = (1 + \xi) p^{\mu} + \frac{M^2}{s(1+\xi)} n^{\mu}$

- Final state particle momenta:
  - $p_2^{\mu} = (1 - \xi) p^{\mu} + \frac{M^2 + \vec{p}_t^2}{s(1 - \xi)} n^{\mu} + \frac{\Delta_{\perp}^{\mu}}{2}$
  - $k^{\mu} = \alpha n^{\mu} + \frac{(\vec{p}_t - \Delta_{t}/2)^2}{\alpha s} p^{\mu} + p_{\perp}^{\mu} - \frac{\Delta_{\perp}^{\mu}}{2}$
  - $p_{\rho}^{\mu} = \alpha_{\rho} n^{\mu} + \frac{(\vec{p}_t + \Delta_{t}/2)^2 + m_{\rho}^2}{\alpha_{\rho} s} p_{\perp}^{\mu} - p_{\perp}^{\mu} - \frac{\Delta_{\perp}^{\mu}}{2}$
Helicity conserving GPDs at twist 2:

\[
\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi}_q \left( -\frac{1}{2} z^- \right) \gamma^+ \psi \left( \frac{1}{2} z^- \right) | p_1, \lambda_1 \rangle = \frac{1}{2P^+} \bar{u}(p_2, \lambda_2) \left[ H^q(x, \xi, t) \gamma^+ + E^q(x, \xi, t) \frac{i\sigma^\alpha + \Delta_\alpha}{2m} \right] u(p_1, \lambda_1)
\]

\[
\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi}_q \left( -\frac{1}{2} z^- \right) \gamma^+ \gamma^5 \psi \left( \frac{1}{2} z^- \right) | p_1, \lambda_1 \rangle = \frac{1}{2P^+} \bar{u}(p_2, \lambda_2) \left[ \tilde{H}^q(x, \xi, t) \gamma^+ \gamma^5 + \tilde{E}^q(x, \xi, t) \frac{\gamma^5 \Delta^+}{2m} \right] u(p_1, \lambda_1)
\]

We will consider the simplest case when $\Delta_\perp = 0$.

In that case and in the forward limit $\xi \to 0$ only the $H^q$ and $\tilde{H}^q$ terms survive.

Helicity conserving (vector) DA at twist 2: longitudinal polarization

\[
\langle 0 | \bar{u}(0) \gamma^\mu u(x) | \rho^0(p, s) \rangle = \frac{p^\mu}{\sqrt{2}} f_\rho \int_0^1 du \ e^{-iu p \cdot x} \phi_\parallel(u)
\]
Non perturbative chiral-odd building blocks

- Helicity flip GPD at twist 2:

\[
\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi}_q \left( -\frac{1}{2} z^- \right) i\sigma^{+i} \psi \left( \frac{1}{2} z^- \right) | p_1, \lambda_1 \rangle
\]

\[
= \frac{1}{2P^+} \bar{u}(p_2, \lambda_2) \left[ H^q_T(x, \xi, t) i\sigma^{+i} + \tilde{H}^q_T(x, \xi, t) \frac{P^+ \Delta^i - \Delta^+ P^i}{M_N^2} \right]
\]

\[
+ E^q_T(x, \xi, t) \frac{\gamma^+ \Delta^i - \Delta^+ \gamma^i}{2M_N} + \tilde{E}^q_T(x, \xi, t) \frac{\gamma^+ P^i - P^+ \gamma^i}{M_N} \right] u(p_1, \lambda_1)
\]

- We will consider the simplest case when \( \Delta_\perp = 0 \).

- In that case and in the forward limit \( \xi \rightarrow 0 \) only the \( H^q_T \) term survives.

- Transverse \( \rho \) DA at twist 2:

\[
\langle 0 | \bar{u}(0) \sigma^{\mu\nu} u(x) | \rho^0(p, s) \rangle = \frac{i}{\sqrt{2}} (\epsilon^\mu_p \rho^\nu - \epsilon^\nu_p \rho^\mu) f_{\rho} \int_0^1 du \ e^{-iu_p \cdot x} \phi_\perp(u)
\]
Models for DAs

Asymptotical DAs

We take the asymptotic form of the (normalized) DAs:

\[
\phi_{\parallel}(z) = 6z(1 - z), \\
\phi_{\perp}(z) = 6z(1 - z).
\]

due to conformal symmetry, \( \mu_F^2 \to \infty \).
Model for GPDs: based on the Double Distribution ansatz

Realistic Parametrization of GPDs

- GPDs can be represented in terms of Double Distributions [Radyushkin]
  
  based on the Schwinger representation of a toy model for GPDs which has the structure of a triangle diagram in scalar $\phi^3$ theory

$$H^q(x, \xi, t = 0) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \; \delta(\beta + \xi\alpha - x) f^q(\beta, \alpha)$$

- ansatz for these Double Distributions [Radyushkin]:

  - chiral-even sector:
    $$f^q(\beta, \alpha, t = 0) = \Pi(\beta, \alpha) q(\beta)\Theta(\beta) - \Pi(-\beta, \alpha) \bar{q}(-\beta) \Theta(-\beta) ,$$
    $$\tilde{f}^q(\beta, \alpha, t = 0) = \Pi(\beta, \alpha) \Delta q(\beta)\Theta(\beta) + \Pi(-\beta, \alpha) \Delta \bar{q}(-\beta) \Theta(-\beta) .$$

  - chiral-odd sector:
    $$f^q_T(\beta, \alpha, t = 0) = \Pi(\beta, \alpha) \delta q(\beta)\Theta(\beta) - \Pi(-\beta, \alpha) \delta \bar{q}(-\beta) \Theta(-\beta) ,$$
    $$\Pi(\beta, \alpha) = \frac{3}{4} \frac{(1-\beta)^2-\alpha^2}{(1-\beta)^3} : \text{profile function}$$

  - simplistic factorized ansatz for the $t$-dependence:
    $$H^q(x, \xi, t) = H^q(x, \xi, t = 0) \times F_H(t)$$
    $$\text{with } F_H(t) = \frac{C^2}{(t-C)^2} : \text{a standard dipole form factor (} C = .71 \text{ GeV)}$$
Model for GPDs: based on the Double Distribution ansatz

Sets of used PDFs

- $q(x)$: unpolarized PDF [GRV-98]
- $\Delta q(x)$: polarized PDF [GRSV-2000]
- $\delta q(x)$: transversity PDF [Anselmino et al.]
Model for GPDs: based on the Double Distribution ansatz

Typical sets of chiral-even GPDs

\[ \xi = 0.1 \leftrightarrow S_{\gamma N} = 20 \text{ GeV}^2 \text{ and } M_{\gamma \rho}^2 = 3.5 \text{ GeV}^2 \]

\[ \frac{1}{2} H_u^{(-)}(x, \xi) \]

\[ \frac{1}{2} H_d^{(-)}(x, \xi) \]

“valence” and “standard”: two GRSV Ansätze for \( \Delta q(x) \)
Model for GPDs: based on the Double Distribution ansatz

Typical sets of chiral-odd GPDs

\[ \xi = 0.1 \leftrightarrow S_{\gamma N} = 20 \text{ GeV}^2 \text{ and } M_{\gamma\rho}^2 = 3.5 \text{ GeV}^2 \]

\[ \frac{1}{2} H_T^{u(-)}(x, \xi) \quad \quad \frac{1}{2} H_T^{d(-)}(x, \xi) \]

“valence” and “standard”: two GRSV Ansätze for \( \Delta q(x) \)
⇒ two Ansätze for \( \delta q(x) \)
Computation of the hard part

20 diagrams to compute

The other half can be deduced by $q \leftrightarrow \bar{q}$ (anti)symmetry

Red diagrams cancel in the chiral-odd case
Final computation

\[ A \propto \int_{-1}^{1} dx \int_{0}^{1} dz \ T(x, \xi, z) \ H(x, \xi, t) \ \Phi_\rho(z) \]

- One performs the \( z \) integration **analytically** using an asymptotic DA \( \propto z(1 - z) \)

- One then plugs our GPD models into the formula and performs the integral w.r.t. \( x \) numerically.

- Differential cross section:

\[
\left. \frac{d\sigma}{dt \ du' \ dM_{\gamma\rho}^2} \right|_{-t = (-t)_{\text{min}}} = \frac{|M|^2}{32 S_{\gamma N}^2 M_{\gamma\rho}^2 (2\pi)^3}.
\]

\( |M|^2 \) = averaged amplitude squared

- Kinematical parameters: \( S_{\gamma N}^2, M_{\gamma\rho}^2 \) and \(-u'\)
**Fully differential cross section**

**Chiral even cross section**

at $-t = (-t)_{\text{min}}$

\[
\frac{d\sigma_{\text{even}}}{dM_{\gamma\rho}^2 d(-u')d(-t)} \quad \text{(nb} \cdot \text{GeV}^{-6})
\]

\begin{align*}
\text{proton} & \\
\text{neutron}
\end{align*}

\[
S_{\gamma N} = 20 \text{ GeV}^2 \\
M_{\gamma\rho}^2 = 3, 4, 5, 6 \text{ GeV}^2
\]

solid: “valence” model
dotted: “standard” model
Fully differential cross section

Chiral odd cross section
at $-t = (-t)_{\text{min}}$

\[
\frac{d\sigma_{\text{odd}}}{dM_{\gamma\rho}^2 d(-u')d(-t)} \text{ (pb} \cdot \text{GeV}^{-6})
\]

proton

"valence" and "standard" models,
each of them with $\pm 2\sigma$ [S. Melis]

\[
S_{\gamma N} = 20 \text{ GeV}^2
\]
\[
M_{\gamma\rho}^2 = 3, 4, 5, 6 \text{ GeV}^2
\]

neutron

"valence" model only
Phase space integration

Evolution of the phase space in \((-t, -u')\) plane

large angle scattering: \(M_{\gamma \rho}^2 \sim -u' \sim -t'\)

in practice: \(-u' > 1 \text{ GeV}^2\) and \(-t' > 1 \text{ GeV}^2\) and \((-t)_{\text{min}} \leq -t \leq .5 \text{ GeV}^2\)

this ensures large \(M_{\gamma \rho}^2\)

example: \(S_{\gamma N} = 20 \text{ GeV}^2\)

\[-u', -t, M_{\gamma \rho}\]
Introduction

Access to GPDs through a 3 body final state

Non-perturbative ingredients

Computation

Results

Conclusion

Single differential cross section

Chiral even cross section

\[ \frac{d\sigma_{\text{even}}}{dM_{\gamma\rho}^2} \text{ (nb} \cdot \text{GeV}^{-2}) \]

\( \begin{array}{c}
\text{proton} \\
\text{neutron}
\end{array} \)

“valence” scenario

\[ S_{\gamma N} \text{ vary in the set } 8, 10, 12, 14, 16, 18, 20 \text{ GeV}^2 \text{ (from left to right)} \]
Single differential cross section

Chiral odd cross section

\[ \frac{d\sigma_{\text{odd}}}{dM_{\gamma\rho}^2} \text{ (pb \cdot GeV}^{-2}) \]

\[ S_{\gamma N} = 20 \text{GeV}^2 \]

Various ansätze for the PDFs $\Delta q$ used to build the GPD $H_T$:

- **dotted curves**: “standard” scenario
- **solid curves**: “valence” scenario
- **deep-blue** and **red** curves: central values
- **light-blue** and **orange**: results with $\pm 2\sigma$. 
Single differential cross section

Chiral odd cross section

\[ \frac{d\sigma_{\text{odd}}}{dM_{\gamma\rho}^2} \text{ (pb} \cdot \text{GeV}^{-2}) \]

proton, “valence” scenario

\( S_{\gamma N} \) vary in the set \( 8, 10, 12, 14, 16, 18, 20 \) GeV\(^2\) (from left to right)
Chiral even cross section

\( \sigma_{even} \) (nb)

- **proton**
  - solid red: “valence” scenario
  - dashed blue: “standard” one

- **neutron**

\( S_{\gamma N}(\text{GeV}^2) \)
Chiral odd cross section

\[ \sigma_{\text{odd}} \text{ (pb)} \]

\[ S_{\gamma N} \text{ (GeV}^2) \]

Solid red: “valence” scenario
Dashed blue: “standard” one
Counting rates for 100 days

example: JLab Hall B

- untagged incoming $\gamma \Rightarrow$ Weizsäcker-Williams distribution

- With an expected luminosity of $\mathcal{L} = 100 \text{ nb}^{-1} \text{s}^{-1}$, for 100 days of run:
  - Chiral even case : $\simeq 3 \ 10^6 \ \rho_L$.
  - Chiral odd case : $\simeq 7 \ 10^3 \ \rho_T$
Effects of an experimental angular restriction for the produced $\gamma$?

Angular distribution of the produced $\gamma$ (chiral-even cross section)

after boosting to the lab frame

\[ \frac{1}{\sigma_{\text{even}}} \frac{d\sigma_{\text{even}}}{d\theta} \]

\[ \begin{array}{c}
\theta \\
0 & 10 & 20 & 30 & 40 \\
\sigma_{\text{even}} & 0.00 & 0.02 & 0.04 & 0.06 & 0.08 & 0.10 \\
\end{array} \]

\[ \begin{array}{c}
\theta \\
0 & 5 & 10 & 15 & 20 & 25 & 30 & 35 \\
\sigma_{\text{even}} & 0.00 & 0.05 & 0.10 & 0.15 & 0.20 \\
\end{array} \]

\[ \begin{array}{c}
\theta \\
0 & 5 & 10 & 15 & 20 & 25 & 30 \\
\sigma_{\text{even}} & 0.00 & 0.05 & 0.10 & 0.15 & 0.20 \\
\end{array} \]

\[ S_{\gamma N} = 10 \text{ GeV}^2 \]

\[ M_{\gamma \rho}^2 = 3, 4 \text{ GeV}^2 \]

\[ J\text{Lab Hall B detector equipped between } 5^\circ \text{ and } 35^\circ \]

\[ \Rightarrow \text{this is safe!} \]
Effects of an experimental angular restriction for the produced $\gamma$?

Angular distribution of the produced $\gamma$ (chiral-odd cross section)

after boosting to the lab frame

\[
\frac{1}{\sigma_{\text{odd}}} \frac{d\sigma_{\text{odd}}}{d\theta}
\]

$S_{\gamma N} = 10 \text{ GeV}^2$

$M_{\gamma \rho}^2 = 3, 4 \text{ GeV}^2$

JLab Hall B detector equipped between $5^\circ$ and $35^\circ$

$\Rightarrow$ this is safe!
Conclusion

- High statistics for the chiral-even component: enough to extract $H$ ($\tilde{H}$?) and test the universality of GPDs.

- In this chiral-even sector: analogy with Timelike Compton Scattering, the $\gamma \rho$ pair playing the role of the $\gamma^*$. 

- Strong dominance of the chiral-even component w.r.t. the chiral-odd one:
  - In principle the separation $\rho_L/\rho_T$ can be performed by an angular analysis of its decay products, but this could be very challenging.
  - Cuts in $\theta_\gamma$ might help.
  - Future: study of polarization observables $\Rightarrow$ sensitive to the interference of these two amplitudes.

- The Bethe Heitler component (outgoing $\gamma$ emitted from the incoming lepton) is:
  - zero for the chiral-odd case
  - suppressed for the chiral-even case.

- Our result can also be applied to electroproduction ($Q^2 \neq 0$) after adding Bethe-Heitler contributions and interferences.

- Possible measurement at JLAB (Hall B, C, D).

- A similar study could be performed at COMPASS. EIC, LHC in UPC?
Chiral-even cross section

### Contribution of $u$ versus $d$

\[
\frac{d\sigma_{\text{even}}}{dM_{\gamma p}^2 d(-u')d(-t)} \text{ (nb} \cdot \text{GeV}^{-6})
\]

$M_{\gamma p}^2 = 4 \text{ GeV}^2$. Both vector and axial GPDs are included.

**Solid**: “valence” model

**Dotted**: “standard” model

- $u$-quark contribution dominates due to the charge effect
- The interference between $u$ and $d$ contributions is important and negative.
Chiral-even cross section

Contribution of vector versus axial amplitudes

\[ \frac{d\sigma_{\text{even}}}{dM_{\gamma\rho}^2 d(-u')d(-t)} \text{ (nb} \cdot \text{GeV}^{-6}) \]

proton

\[ M_{\gamma\rho}^2 = 4 \text{ GeV}^2 \]. Both \( u \) and \( d \) quark contributions are included.

neutron

vector + axial amplitudes / vector amplitude / axial amplitude

- solid: “valence” model
- dotted: “standard” model

- dominance of the vector GPD contributions
- no interference between the vector and axial amplitudes
DZIĘKUJĘ BARDZO ZA UWAGĘ

THANK YOU FOR YOUR ATTENTION