Lessons from hydro at large orders

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Introduction

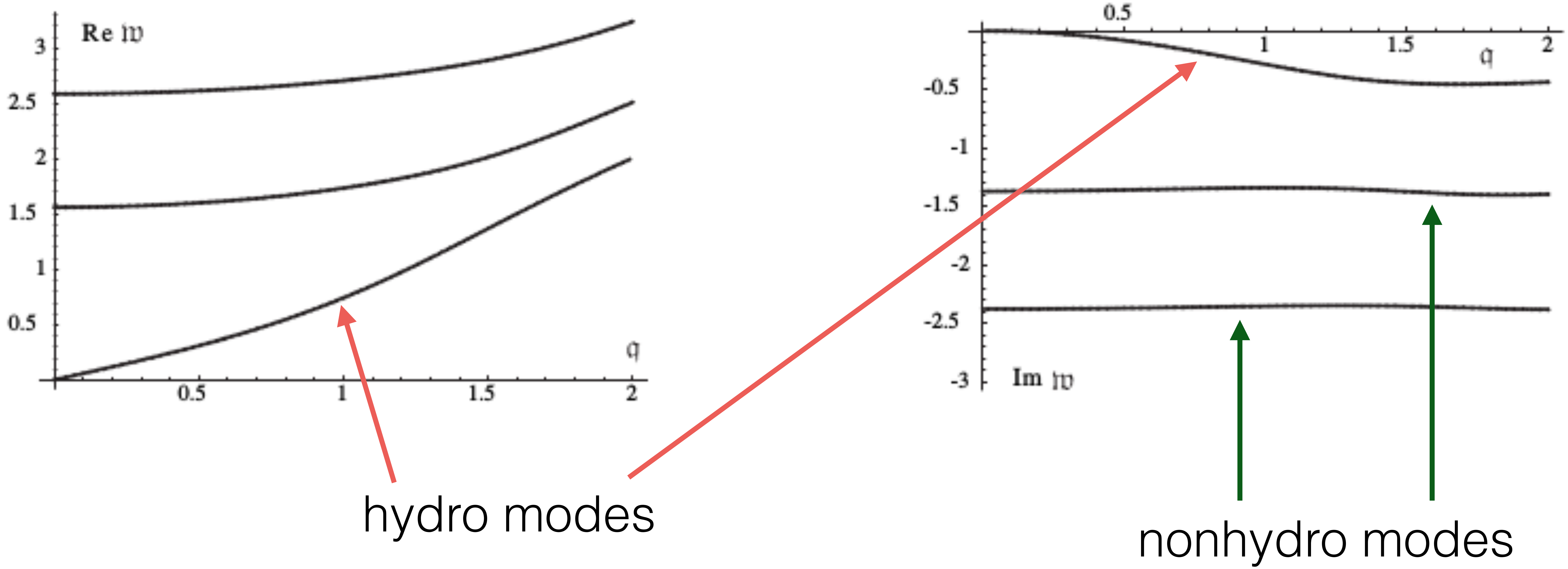
- **Hydro is doing well** as a pheno description of QGP@RHIC, @LHC
- Task: **explain why** and turn it into an effective theory **matching QCD**
- Useful input from **first principles calculations** using AdS/CFT:
  A. Complete 2nd order theory (**BRSSS** replacing MIS)
  B. Hydro works early ("**hydrodynamization**")
  C. Gradient expansions are **asymptotic**
- **What about QCD?** - Recent progress based on kinetic theory

Strong coupling picture from AdS/CFT

- Quasinormal modes $\delta\Phi \sim \exp\left(-i(\omega t - \vec{k}\cdot\vec{x})\right)$

depends on k

- Damped when $Im(\omega) < 0$



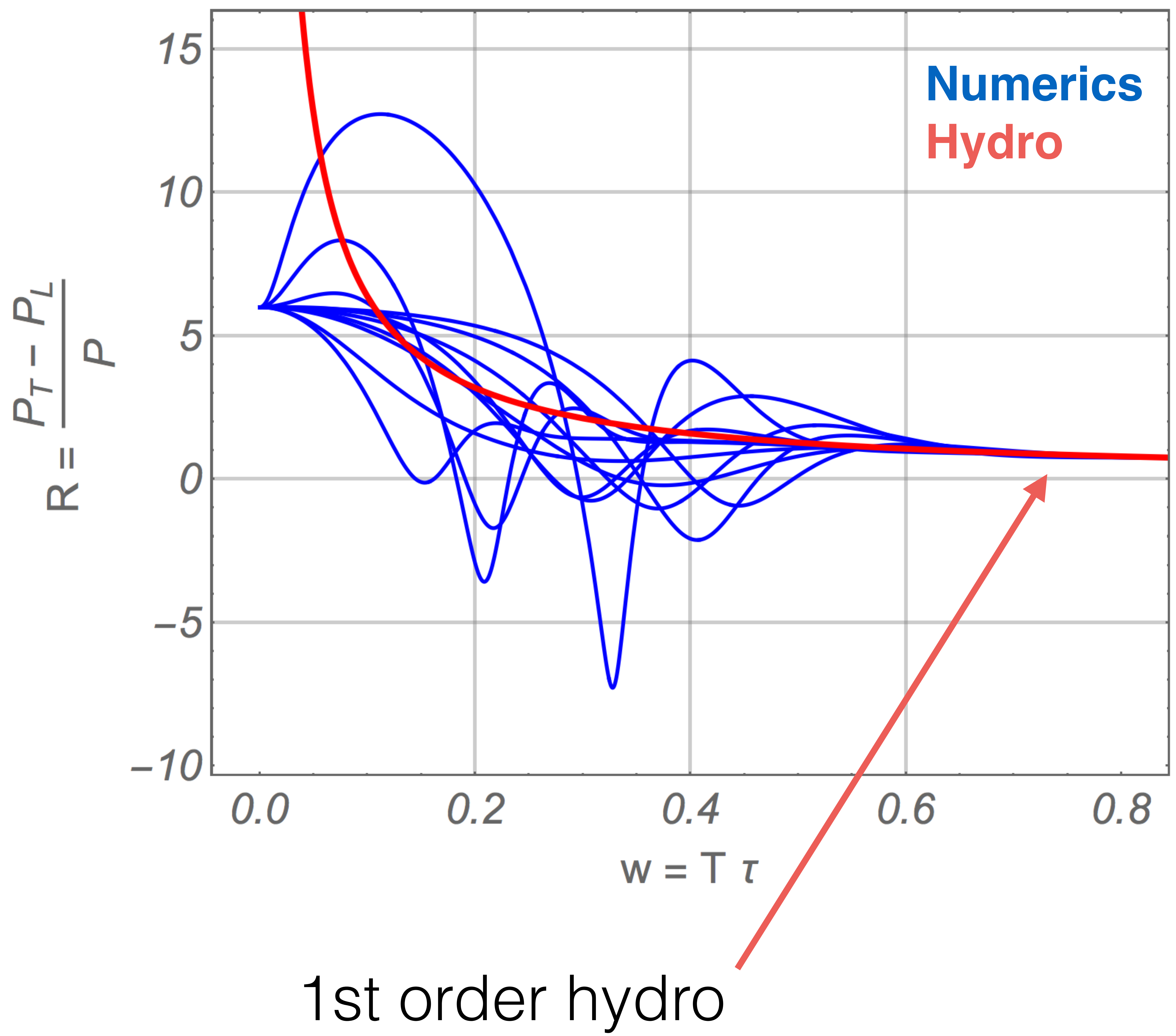
[Kovtun, Starinets hepth/0506184]

Hydrodynamization from holography

- Hydro works great even at large pressure anisotropies (~ 60%).
- Effects of nonhydro modes are plainly visible at small times
- “Hydrodynamization” for

$$\tau T \sim 0.7$$

- Applicability of hydro determined by decay nonhydro modes



[Heller et al.1302.0697]
[Jankowski et al.1411.1969]

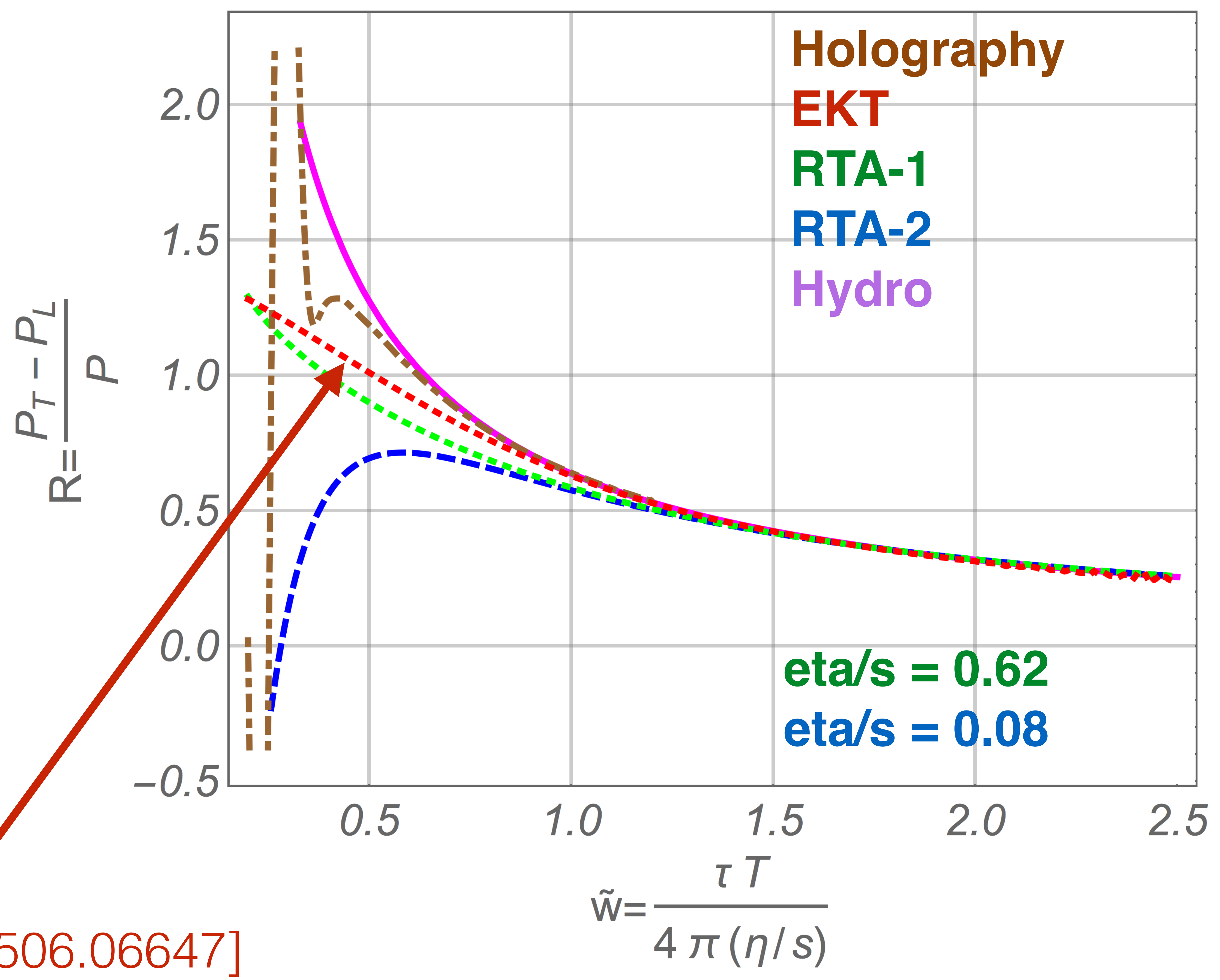
Hydrodynamization from kinetic theory

Boltzmann eqn, 2 collision kernels:

- Relaxation time approximation
- Effective kinetic theory (AMY)

Note:

- Hydrodynamization
- Boring at early time



[Kurkela et al.1506.06647]

[Heller et al.1609.04803]

Hydrodynamic theories

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}(\mathcal{E})(g^{\mu\nu} + u^\mu u^\nu) + \Pi^{\mu\nu}$$

$$\nabla_\alpha T^{\alpha\beta} = 0$$

SST: needed to describe dissipation

- Perfect fluid: $\Pi^{\mu\nu} = 0$

acausal

nohydro modes ensure causality when $T\tau_\Pi \geq 2\eta/s$

- Navier Stokes: $\Pi^{\mu\nu} = -\eta(\mathcal{E})\sigma^{\mu\nu}$
- Mueller; Israel & Stewart; BRSSS: $(\tau_\Pi \mathcal{D} + 1)\,\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \ldots$
- H+QNM: $\left((\frac{1}{T}\mathcal{D})^2 + 2\omega_I \frac{1}{T}\mathcal{D} + |\omega|^2 \right) \Pi^{\mu\nu} = -\eta|\omega|^2\sigma^{\mu\nu} - c_\sigma \frac{1}{T}\mathcal{D}\,(\eta\sigma^{\mu\nu}) + \ldots$
- Anisotropic hydrodynamics

all coincide at late times, differ at early times

$T \sim \mathcal{E}^{1/4}$

The gradient expansion

This works, because the SST can be expressed as a formal **infinite series**

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \tau_\Pi \mathcal{D}(\eta\sigma^{\mu\nu}) + \ldots$$

The relaxation time appears as a second order transport coefficient

whose form is fixed by symmetries, and it

- **defines** what we mean by transport coeffficients
- allows **comparison** between different hydrodynamic theories
- allows **matching** phenomenological and microscopic descriptions

$$\langle T^{\mu\nu}\rangle = \mathcal{E}u^\mu u^\nu + \mathcal{P}(g^{\mu\nu} + u^\mu u^\nu) - \eta\sigma^{\mu\nu} + \tau_\Pi \mathcal{D}(\eta\sigma^{\mu\nu}) + \lambda_1 \left(\sigma^2\right)^{\mu\nu} + \ldots$$

Calculated explicitly in some examples

Definition of u: $\langle T^\nu_\mu\rangle u^\mu = -\mathcal{E}u^\nu$

Theoretical laboratory: Bjorken flow

Energy-momentum tensor:

$$\langle T^{\mu}_{\nu} \rangle = \mathrm{diag}(-\mathcal{E}, \mathcal{P}_L, \mathcal{P}_T, \mathcal{P}_T)$$

$\mathcal{E}(\tau)$

$$\mathcal{P}_L = -\mathcal{E} - \tau\dot{\mathcal{E}}\ , \quad \mathcal{P}_T = \mathcal{E} + \frac{1}{2}\tau\dot{\mathcal{E}}$$

Independent of initial conditions

Large proper-time (gradient) expansion:

$$T(\tau) = \frac{\Lambda}{(\Lambda\tau)^{1/3}}\left(1 + \frac{t_1}{(\Lambda\tau)^{2/3}} + \frac{t_2}{(\Lambda\tau)^{4/3}} + \dots\right)$$

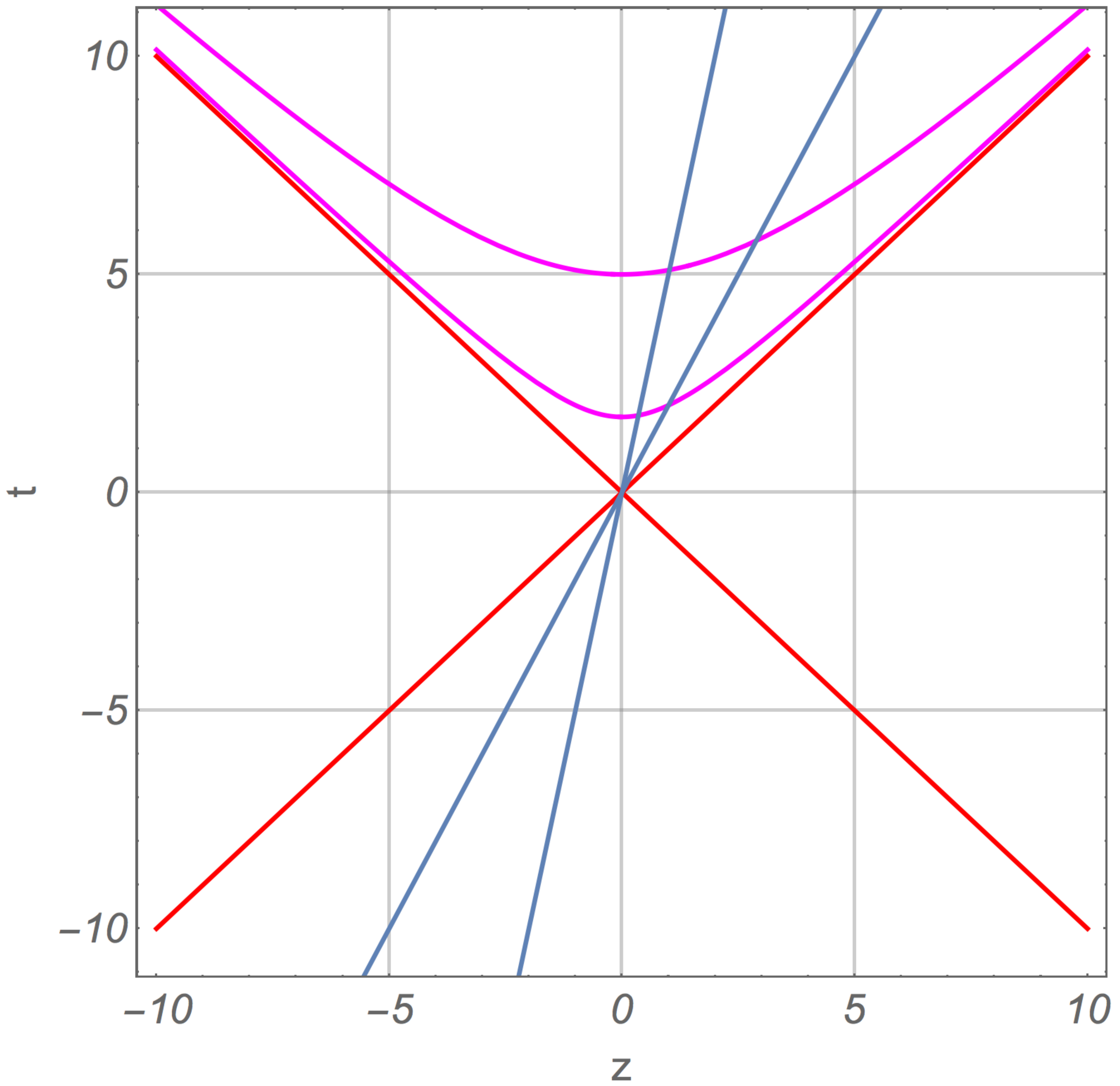
Remnant of initial conditions

Dimensionless variables:

$$w \equiv \tau T, \quad \mathcal{R} \equiv \frac{\mathcal{P}_T - \mathcal{P}_L}{\mathcal{P}}$$

$$f \equiv \frac{2}{3}\left(1 + \frac{\mathcal{R}}{12}\right) = \sum_{n=0}^{\infty} f_n w^{-n}$$

$$t = \tau\cosh\lambda\ , \quad z = \tau\sinh\lambda$$



Large order behaviour

The gradient expansion coefficients have been computed in some **microscopic models** with the result

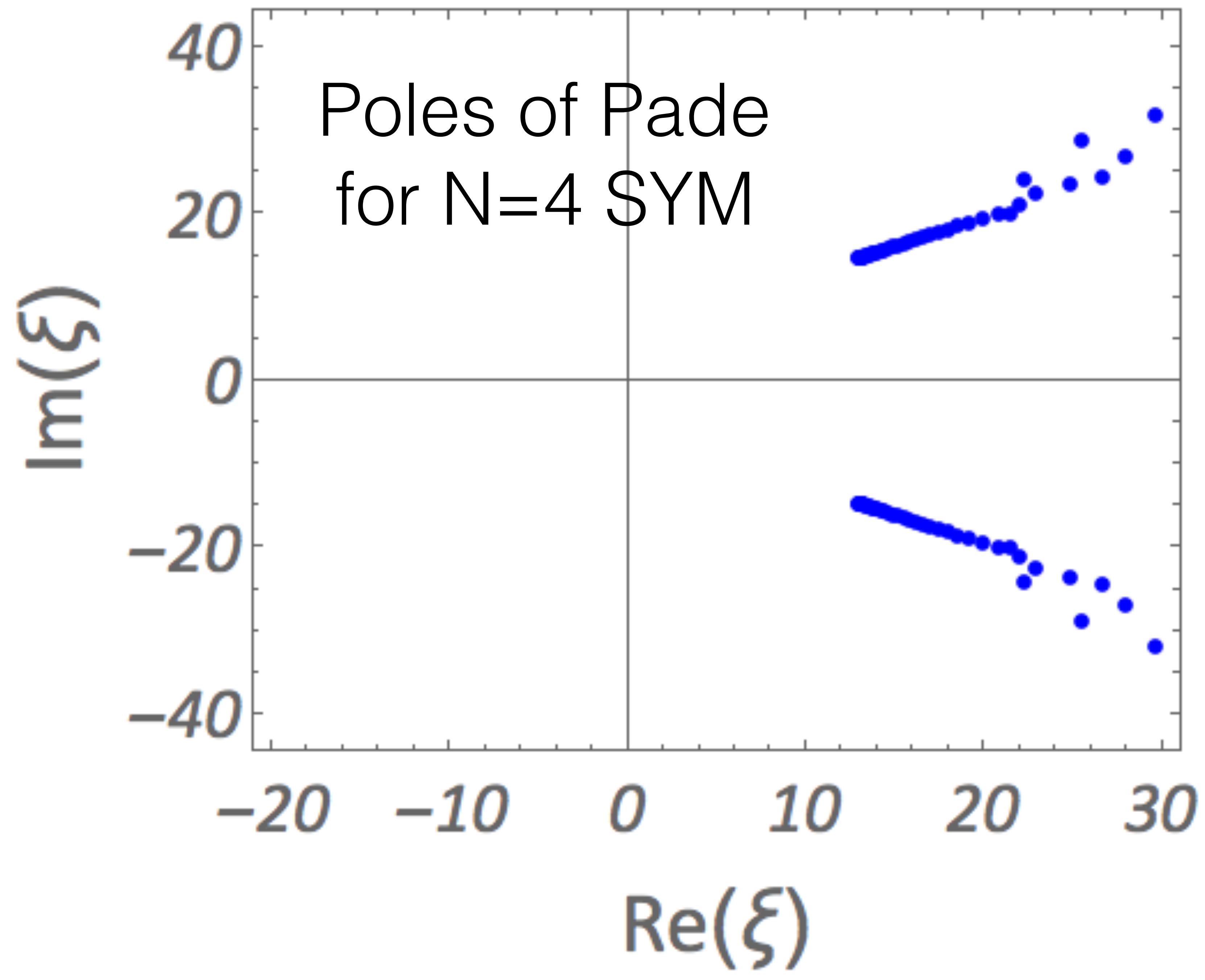
$$f_n \sim n!$$

Similar calculations in **hydrodynamics also** lead to divergent series

The singularities of the analytic continuation of the Borel transform

$$f_B(\xi) = \sum_{n=0}^{\infty} \frac{f_n}{n!} \xi^n$$

contain **information about nonhydro modes** of the system.



[Heller et al.1302.0697]

Bjorken flow in BRSSS

Evolution equation

$$w f f' + 4f^2 + \left(\frac{w}{C_{\tau\Pi}} - \frac{16}{3}\right) f - \frac{2w}{3C_{\tau\Pi}} - \frac{4C_\eta}{9C_{\tau\Pi}} + \frac{16}{9} = 0$$

where

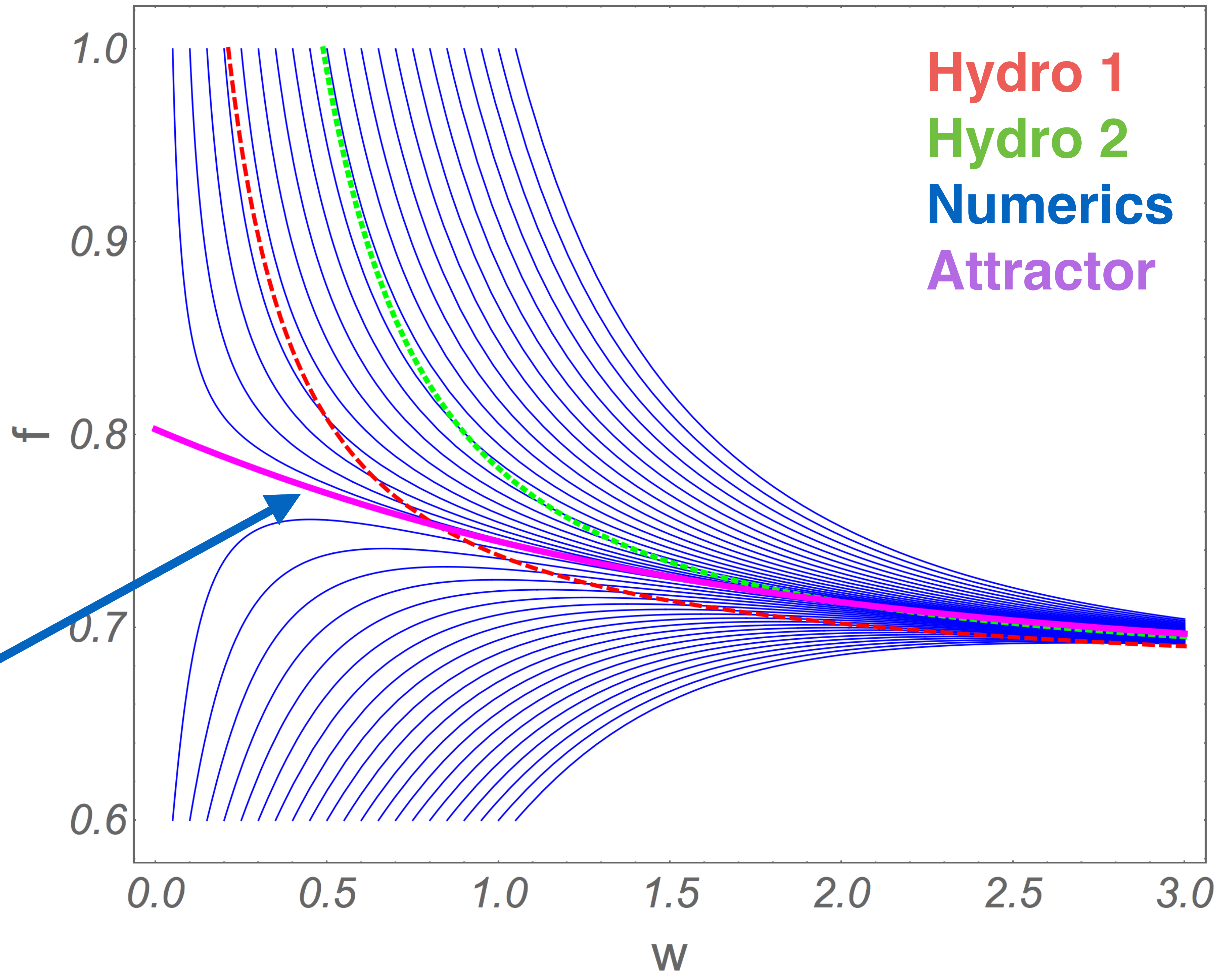
$$C_{\tau\Pi} = T\tau_\Pi, \quad C_\eta = \eta/s$$

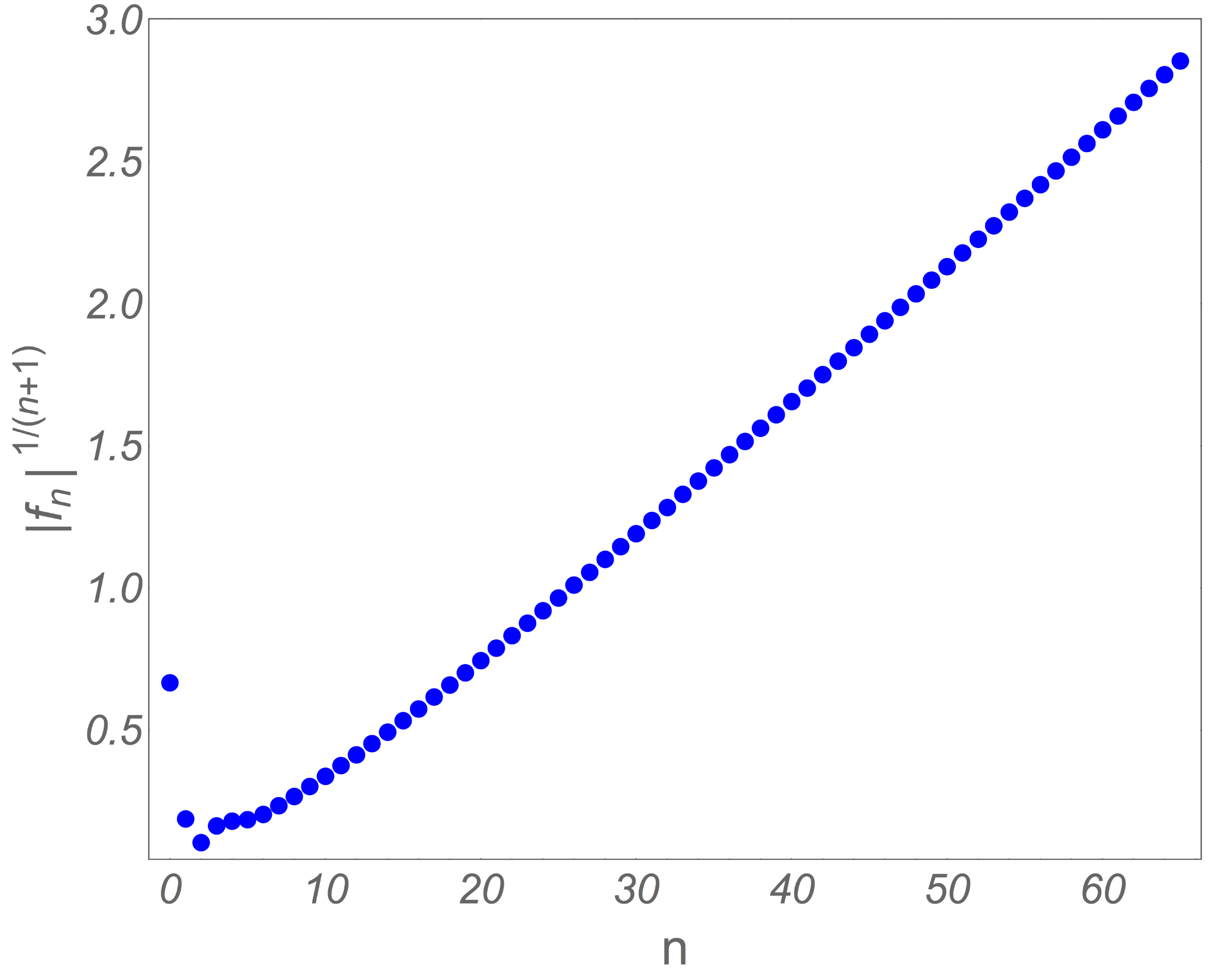
Finite order hydrodynamics:

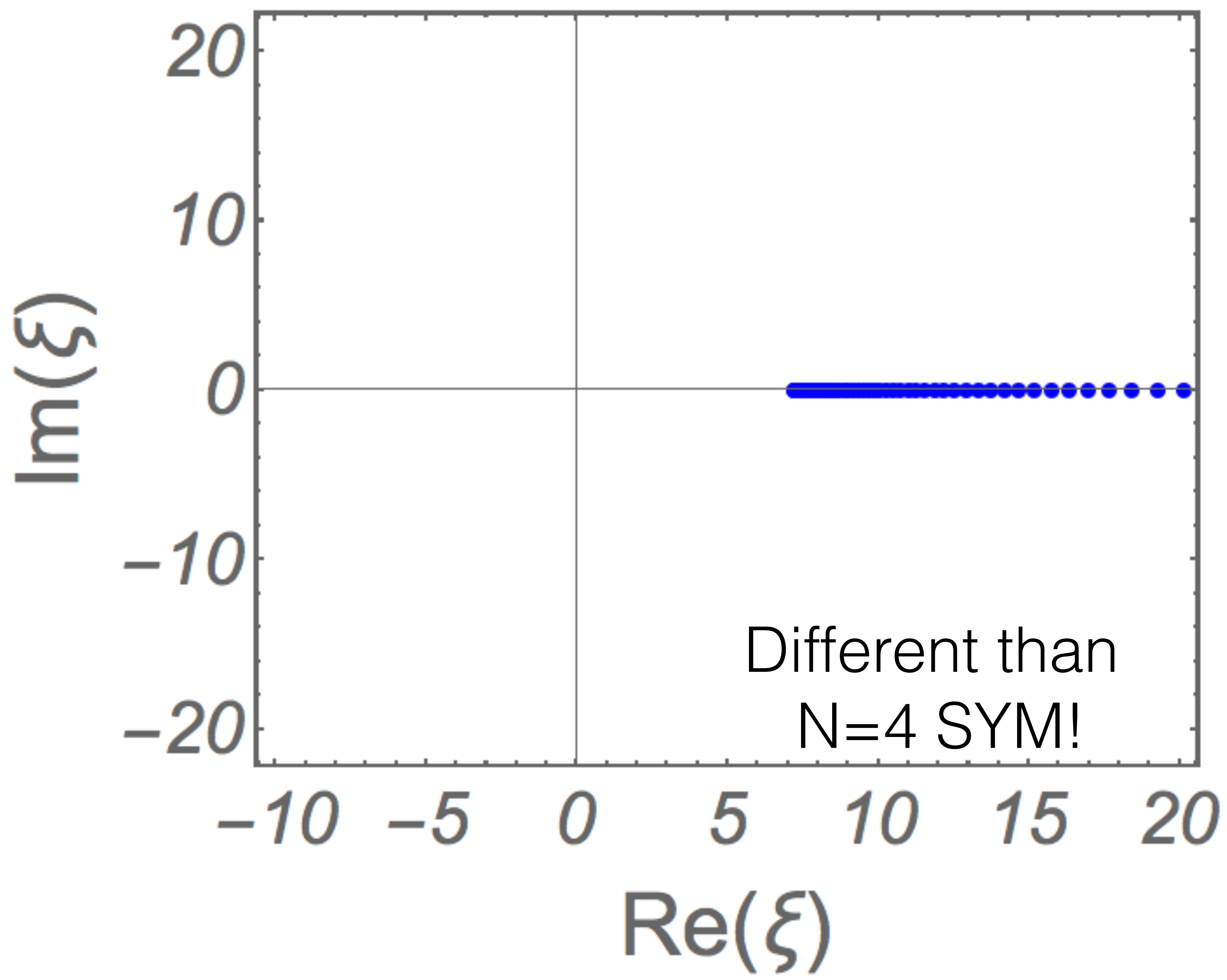
$$f(w) = \boxed{\boxed{\frac{2}{3} + \frac{4C_\eta}{9w}} + \frac{8C_\eta C_{\tau\Pi}}{27w^2}} + \dots$$

Attractor = “resummed hydro”?

[Heller, MS 1503.07514]







- The series is **asymptotic**
- Single purely damped nonhydro mode, decay rate **given by cut location**
- Resummation ambiguity resolved by **resurgence**
- Similar picture for the **kinetic theory** RTA model (with subleading cuts)

Summary

- Holography has contributed to advancing the formulation and interpretation of relativistic hydrodynamics
- Relativistic hydrodynamic theories include nonhydrodynamic modes which serve as a **regulator** for causality
- Information about nonhydrodynamic modes is encoded in the **large order behaviour** of the gradient expansion
- In principle, hydrodynamic theories can be **engineered** to match the gradient expansion and the nonhydrodynamic sector of a given microscopic theory