THERMAL MODIFICATIONS OF THE SOFT DIJET FUNCTION

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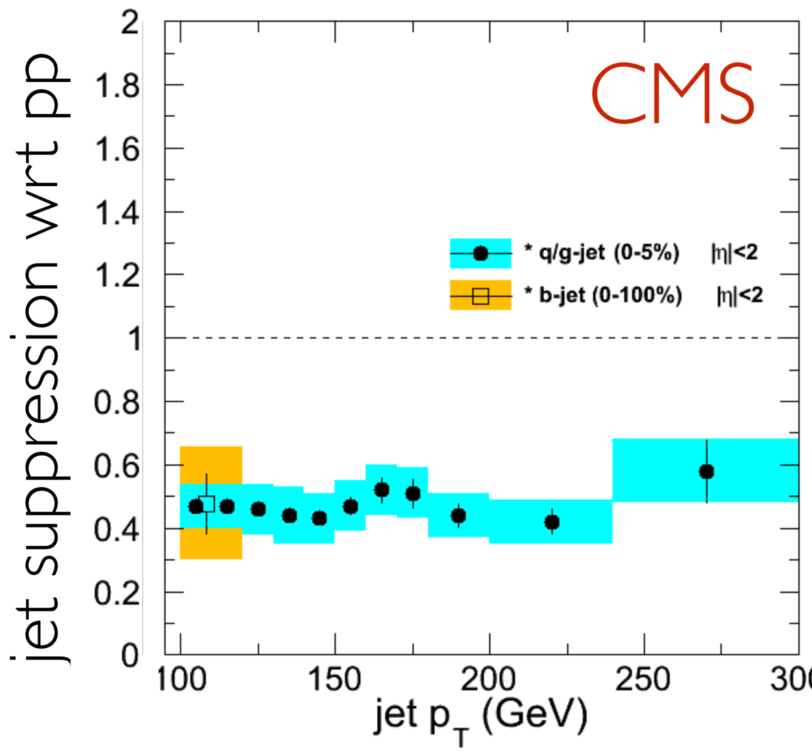
Various Faces of QCD

3rd Symposium of the Division for Physics of Fundamental Interactions of the Polish Physical Society

JETS IN HEAVY-ION COLLISIONS

- strong modifications
  - jet yields
  - dijet energy imbalance
  - intra-jet structure
  - correlations



- hope: can be used as probes of the medium (jet tomography)
- transport coefficient $\hat{q}$ and geometry L

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interface of jets w/ medium: a multi-scale problem!

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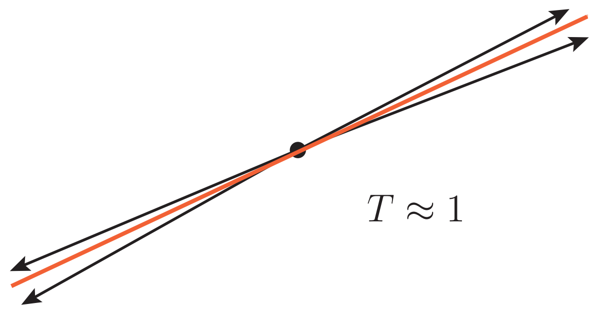
interface of jets w/ medium: a multi-scale problem!

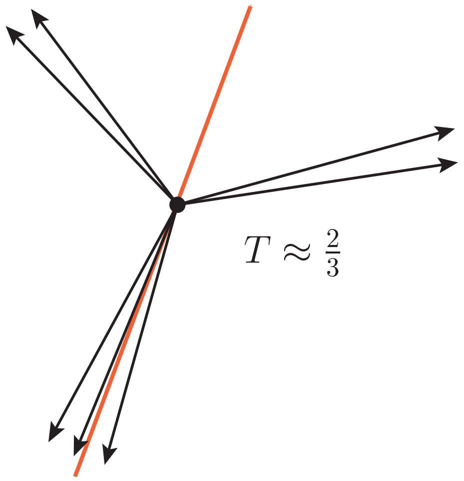
top-down approach: medium effects in presence of jets!

EVENT-SHAPE OBSERVABLES

Thrust/Jet mass

$$t \equiv 1 - T \simeq \frac{M_R^2}{Q^2} + \frac{M_L^2}{Q^2}$$





- consider for the moment e+e- collisions
  - next step: in the presence of a thermal bath
- $t \ll 1$: two narrow jets propagating through a cloud a soft gluons
- how is the energy flow modified?

MOTIVATION

- introduction of non-perturbative effects is important in the context of strong-coupling determination
- universality of "shape function" for many event shapes
- soft gluon flow at large angles
- medium:
  - how does sensitivity to medium scales enter from first principles?
  - EFT approach: matching
- approach: first principle expansion in g (resummation)

THE ROLE OF POWER CORRECTIONS

$$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma}{dt}\bigg|_{\mathbf{PT}} = \sigma_0\left(\alpha_s(Q), \ln t, \frac{1}{Qt}\right) + \mathcal{O}\left(\frac{1}{Q^2 t}\right)$$

$$\frac{1}{\sigma_{\mathrm{tot}}(t)}\frac{\mathrm{d}\sigma}{\mathrm{d}t} \simeq \int_0^{Qt} \mathrm{d}\varepsilon\, f(\varepsilon;\mu)\frac{\mathrm{d}\sigma_{\mathrm{PT}}(t-\varepsilon/Q;\mu)}{\mathrm{d}t}$$

- σ_0 resums PT
  - $\alpha_s^n \ln^m t/t$ + power corrections $1/(Qt)^k$
- end-point region: jet mass^2 ~ $Q^2 t \gg \Lambda_{QCD}^2$
  - Q-independent shape function
  - first mom: shift of PT distribution
  - higher mom: hemisphere correlations

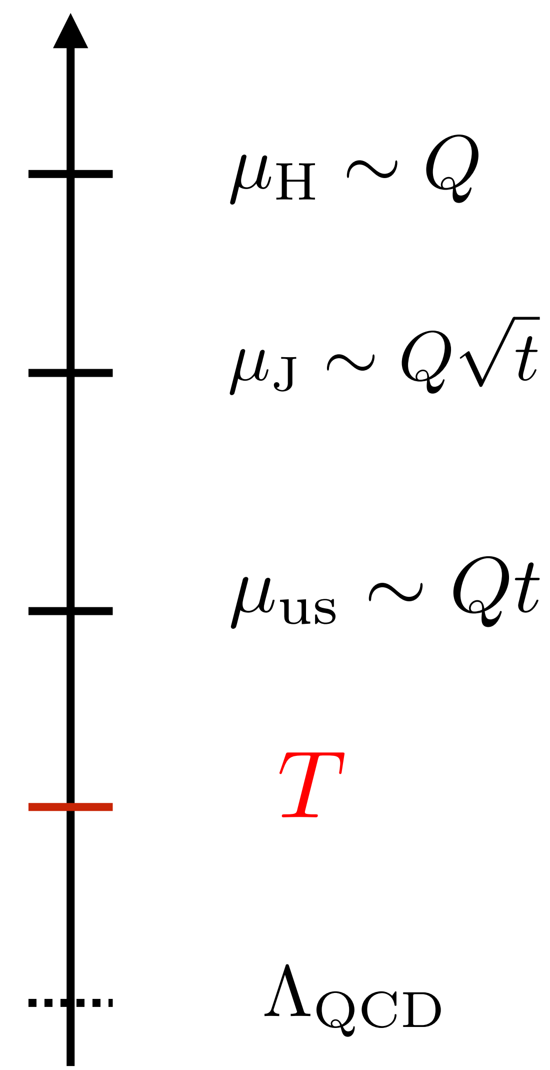
$$f(\varepsilon_L,\varepsilon_R) = f(\varepsilon_R)\, f(\varepsilon_L) + \Delta f(\varepsilon_L,\varepsilon_R)$$

Webber PLB 339 (1994) 148
Manohar, Wise PLB 344 (1995) 407
Korchemsky, Sterman NPB 437 (1995) 415
Korchemsky, Sterman NPB 555 (1995) 335
Korchemsky hep-ph/9806537
Korchemsky, Tafat JHEP 0010 (2000) 010
Dokshitzer, Webber PLB 352 (1995) 451
Dokshitzer, Webber PLB 404 (1997) 321

Hierarchy of scales

$$\frac{1}{\sigma_0}\frac{\mathrm{d}^2\sigma^{\mathrm{dijet}}}{\mathrm{d}M_1^2\,\mathrm{d}M_2^2} = H_Q(Q,\mu)\int_{-\infty}^{\infty}\mathrm{d}l^+\mathrm{d}l^- J_1(M_1^2 - Ql^+,\mu)J_2(M_2^2 - Ql^-,\mu)S(l^+,l^-,\mu)$$

- factorisation of the dijet cross-section
  - hard cross section: Q
  - jet mass: $M_{R,L} \sim Qt^{1/2}$
  - energy of soft gluons $\sim Qt$
  - *new:* temperature $T > \Lambda_{QCD}$
- physically: soft gluons cannot resolve the inner structure of jets
  - jets = Wilson lines
- independence of the hard scale of the collision



The soft function

$$S(l^+, l^-) = \frac{1}{N_c} \mathrm{tr} \left\langle \bar{\mathcal{T}}[\bar{Y}_{\bar{n}}^\dagger(0) Y_n^\dagger(0)]\, \delta\left(l^+ - \mathbb{P}_\mathrm{R}\right) \delta\left(l^- - \mathbb{P}_\mathrm{L}\right)\, \mathcal{T}[Y_n(0) \bar{Y}_{\bar{n}}(0)] \right\rangle$$

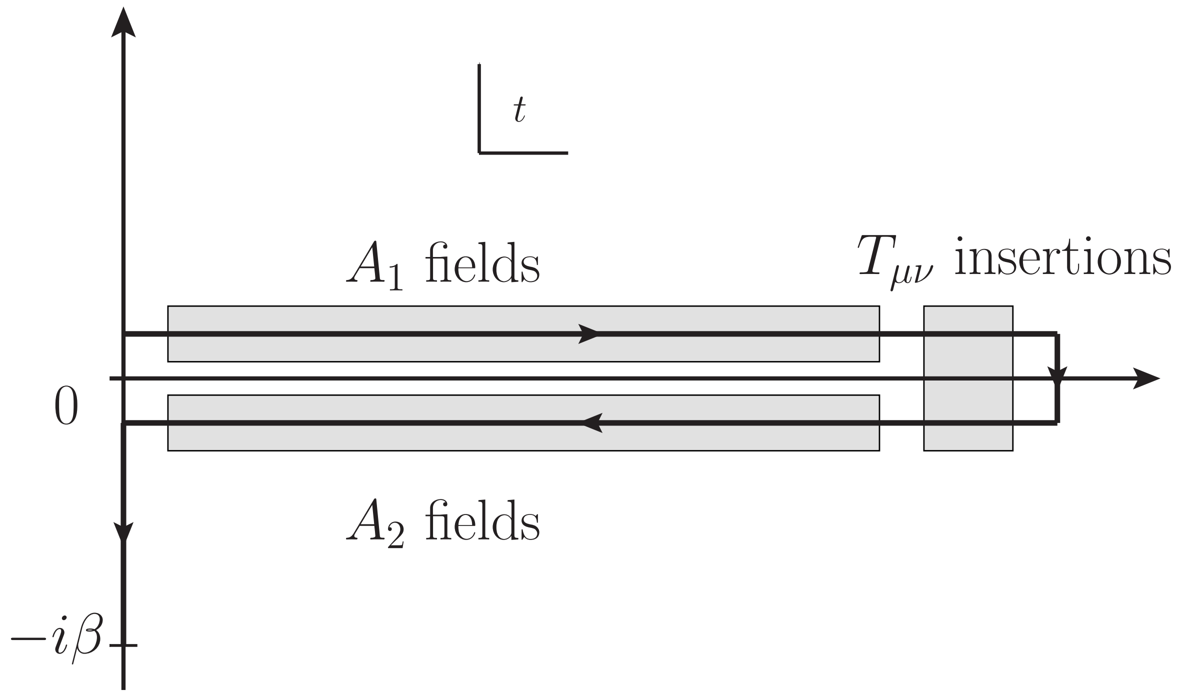
$$\mathbb{P}_\mathrm{R} |X_{us}\rangle = \sum_{i \in R} q_i^+ |X_{us}\rangle$$

$$\mathbb{P}_\mathrm{L} |X_{us}\rangle = \sum_{i \in L} q_i^- |X_{us}\rangle$$

- hemisphere soft function: most general object
- permits a moment expansion (à la OPE)
- left/right soft momentum operators defined with general energy flow operator

Belitsky PLB 442 (1998) 307
Korchemsky, Sterman NPB 555 (1999) 335
Belitsky, Korchemsky, Sterman PLB 515 (2001) 197

SCHWINGER-KELDYSH FORMALISM



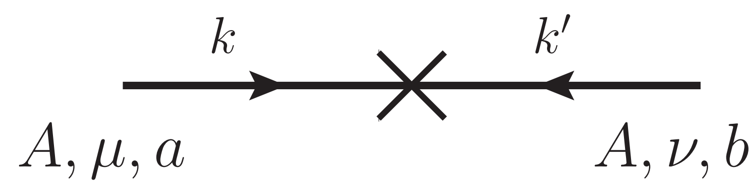
$$\mathcal{A} = \frac{1}{2}(A_1 + A_2)$$

$$\eta = A_1 - A_2$$

- double the field content
  - time ordered fields: amplitude
  - anti-time ordered fields: c.c.
- time-contour ordering
- calculation of weighted cross-sections
- Keldysh (ra) basis: classical correspondence
  - implementation of medium effects (TFT) straightforward

Belitsky PLB 442 (1998) 307
Belitsky, Korchemsky, Sterman PLB 515 (2001) 197
KT, Casalderrey-Solana, Pablos (2016, in preparation)

Definition of the detector



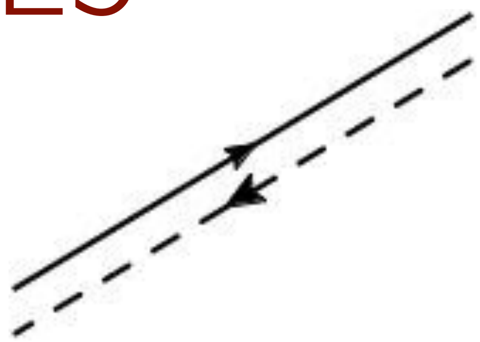
$$\mathcal{E}(\boldsymbol{e}) = \int_{-\infty}^{\infty} \mathrm{d}x_- \lim_{x_+ \to \infty} x_+^2 \, \frac{\bar{e}_\mu \bar{e}_\nu}{e \cdot \bar{e}} T^{\mu\nu}(x_+ e + x_- \bar{e})$$

- what are the quasi-particles that propagate in the presence of background?
- free stress-energy tensor: proportional to in-coming fields
- picks up positive frequencies on the forward light-cone
  - cut procedure

Sveshnikov, Tkachev PLB 382 (1996) 403
Cherzor, Sveshnikov hep-ph/9710349
Korchemsky, Oderda, Sterman hep-ph/9708346
Korchemsky, Sterman NPB 555 (1999) 335
Belitsky, Korchemsky, Sterman PLB 515 (2001) 197
Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov NPB 884 (2014) 305

Expansion of Wilson lines

$$U_n(0) = \mathcal{P}_c \exp\left\{ ig \int_0^\infty \mathrm{d}t \left[n \cdot A_1^a(nt) - n \cdot A_2^a(nt) \right] T^a \right\}$$



For dijets: $$W_{n\bar{n}}(0) = N_c^{-1} \mathrm{tr}\{ \bar{U}^T_{\bar{n}}(0)\, U_n(0) \}$$

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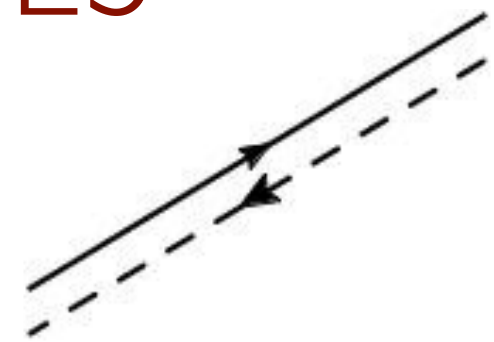
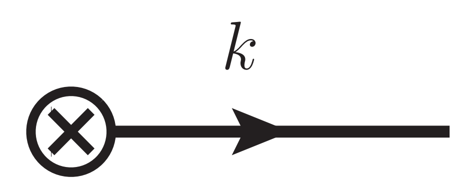


For dijets: $\quad W_{n\bar{n}}(0) = N_c^{-1} \mathrm{tr}\{ \bar{U}_{\bar{n}}^T(0)\, U_n(0) \}$

$$W_{n\bar{n}}^{(2)}(0) = (ig)^2 \frac{\delta^{ab}}{2^2 N_c} \int_0^\infty \mathrm{d}t_1 \mathrm{d}t_1 \boxed{\big(\eta_n^a(t_1) - \eta_{\bar{n}}^a(t_2)\big)} \big(\eta_n^a(t_2) - \eta_{\bar{n}}^a(t_2)\big)$$



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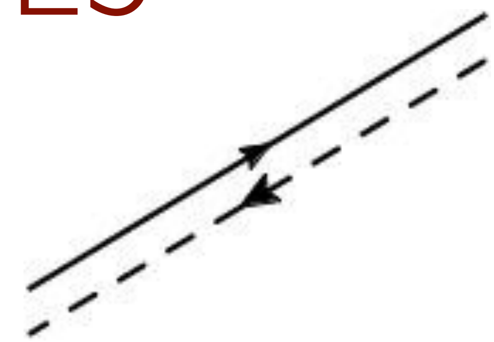
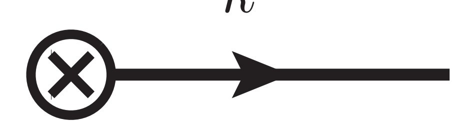
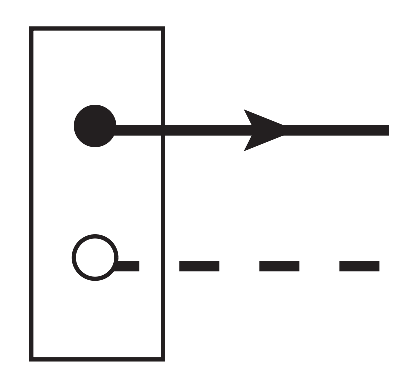


For dijets: $$W_{n\bar n}(0) = N_c^{-1} \mathrm{tr}\{ \bar U_{\bar n}^T(0)\, U_n(0) \}$$

$$W^{(2)}_{n\bar n}(0) = (ig)^2 \frac{\delta^{ab}}{2^2 N_c} \int_0^\infty \mathrm{d}t_1 \mathrm{d}t_1 \boxed{\left(\eta_n^a(t_1) - \eta_{\bar n}^a(t_2)\right)} \left(\eta_n^a(t_2) - \eta_{\bar n}^a(t_2)\right)$$



$$W^{(3)}_{n\bar n}(0) = (ig)^3 \int_0^\infty \mathrm{d}t_1 \mathrm{d}t_2 \mathrm{d}t_3 \Big[\frac{if^{abc}}{N_c} \boxed{\frac{1}{2}\Theta_{12}\left(\eta_n^a(t_1)\mathcal{A}_n^b(t_2) - \eta_{\bar n}^a(t_1)\mathcal{A}_{\bar n}^b(t_2)\right)} \left(\eta_n^c(t_3) - \eta_{\bar n}^c(t_3)\right)$$
$$+ \frac{d^{abc}}{N_c}\frac{1}{4!} \left(\eta_n^a(t_1) - \eta_{\bar n}^a(t_1)\right)\left(\eta_n^b(t_2) - \eta_{\bar n}^b(t_2)\right)\left(\eta_n^c(t_3) - \eta_{\bar n}^c(t_3)\right) \Big],$$



…interesting structure emerges!

WILSON LINES: A NOVEL RECURSION

KT, Casalderrey-Solana, Pablos (2016, in preparation)

$$U_n(0) = \mathcal{P}_c \exp\left\{ ig \int_0^\infty \mathrm{d}t \, [n \cdot A_1^a(nt) - n \cdot A_2^a(nt)] \, T^a \right\}$$

$$U_n(0) = \sum_k \frac{(ig)^k}{k!} \prod_{j=1}^{k} \int \mathrm{d}t_j \, u_n^{(k)}(t_1, \ldots, t_k)$$

Time-ordered contribution at order *n*:

$$u^{(n)} = \sum_{\substack{i_1, \ldots, i_n = 1 \\ i_1 \neq \ldots \neq i_n}}^{n} 2\,\Theta_{i_1,\ldots,i_n} T_{i_1,\ldots,i_n}$$

$$T_{1,\ldots,n} = T_{1,\ldots,n-1} \otimes \mathbb{K}_n$$

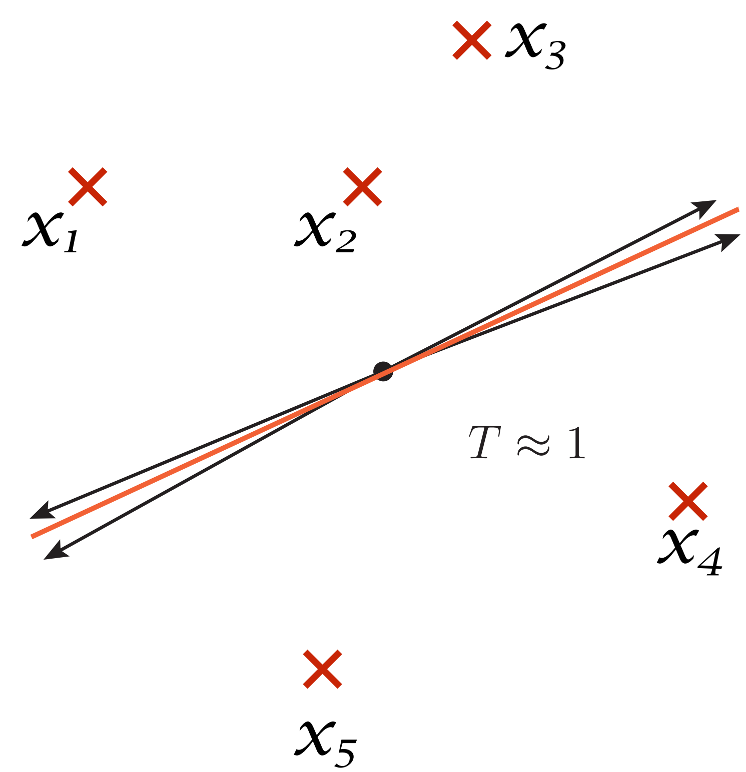
"rotation" operator $\bullet \otimes \mathbb{K}_n \equiv [\bullet, \mathcal{A}_i] + \{\bullet, \eta_i/2\}$

with initial condition: $T_1 = \eta_1/2$

Correlator definition

$$C^{\mu_1\nu_1\dots\mu_n\nu_n}(x_1,\dots,x_n) = \langle \mathcal{T}_c\{T^{\mu_1\nu_1}_{\rm SK}(x_1)\dots T^{\mu_n\nu_n}_{\rm SK}(x_n) W_{n\bar n}(0)\}\rangle$$

- study the correlation of stress-energy insertions — all cumulant structure
- allows to specify what modes reach the detector
- systematic expansion in g



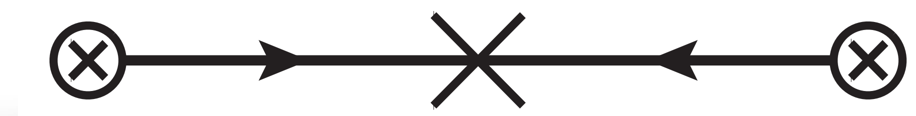
$$\int_0^\infty \mathrm{d}l^+ \int_0^\infty \mathrm{d}l^-\, {l^+}^N {l^-}^M S\left(l^+, l^-\right) \equiv \langle {\mathbb{P}_{\rm R}}^N {\mathbb{P}_{\rm L}}^M \rangle$$

$$= \int \prod_{j=1}^{N+M} \mathrm{d}^3 \boldsymbol{e}_j \delta(\boldsymbol{e}_j^2 - 1) \prod_{n=1}^{N} w_{\rm R}(\boldsymbol{e}_n) \prod_{m=1}^{M} w_{\rm L}(\boldsymbol{e}_m) \langle \mathcal{E}(\boldsymbol{e}_1)\dots\mathcal{E}(\boldsymbol{e}_N)\rangle_Y$$

$$\mathbb{P}_{\rm R/L} = \frac{1}{2}\int_{-1}^{1} \mathrm{d}\cos\theta \int_0^{2\pi} \mathrm{d}\varphi\, w_{\rm R/L}(\theta)\, \mathcal{E}(\theta,\varphi)$$

Belitsky, Korchemsky, Sterman PLB 515 (2001) 197

LEADING ORDER: FIRST MOMENT



- only vacuum contribution
  - easy in ra basis!
- PT: $\rho(k^2)=\Gamma_{cusp}(k^2)/k^2$
  - including higher-order correction to get running α_s
  - rapidity independent density
- IR: substitute by NP density
- UV sensitivity

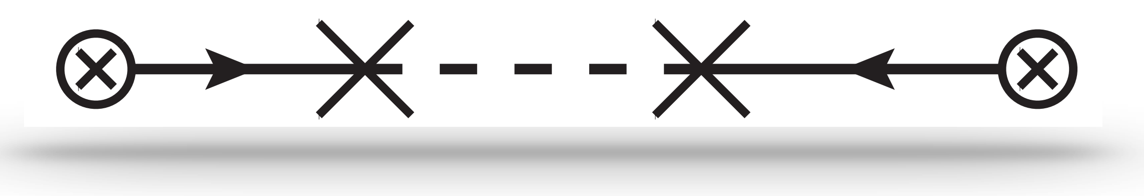
$$\mathcal{G}(\boldsymbol{e}) = \frac{1}{4\pi}\frac{1}{\sin^3\theta}\int_0^{\mu^2} \mathrm{d}\boldsymbol{k}^2\,|\boldsymbol{k}|\rho(\boldsymbol{k}^2)$$

$$\Gamma_{\mathrm{cusp}}(\boldsymbol{k}^2) = \frac{\alpha_s(\boldsymbol{k}^2)C_F}{\pi}$$

SECOND MOMENT: MEDIUM

$$\mathcal{G}(\boldsymbol{e}_1, \boldsymbol{e}_2) = \delta^{(2)}(\boldsymbol{e}_1 - \boldsymbol{e}_2) \frac{1}{\pi} \frac{1}{\sin^4 \theta} \int_0^{\mu^2} \mathrm{d}\boldsymbol{k}^2 \, |\boldsymbol{k}|^2 \rho(\boldsymbol{k}^2) \, \coth\left(\frac{|\boldsymbol{k}|}{2\,T \sin\theta}\right)$$

- first medium contribution at g^2
  - on-shell thermal fluctuations
- rapidity dependence: effective temperature $T_{\mathrm{eff}}(\eta) = T/\cosh(\eta)$
- no correlation between hemispheres
  - expected to appear at $\mathcal{O}(g^4)$



KT, Casalderrey-Solana (2016, in preparation)

Moment resummation

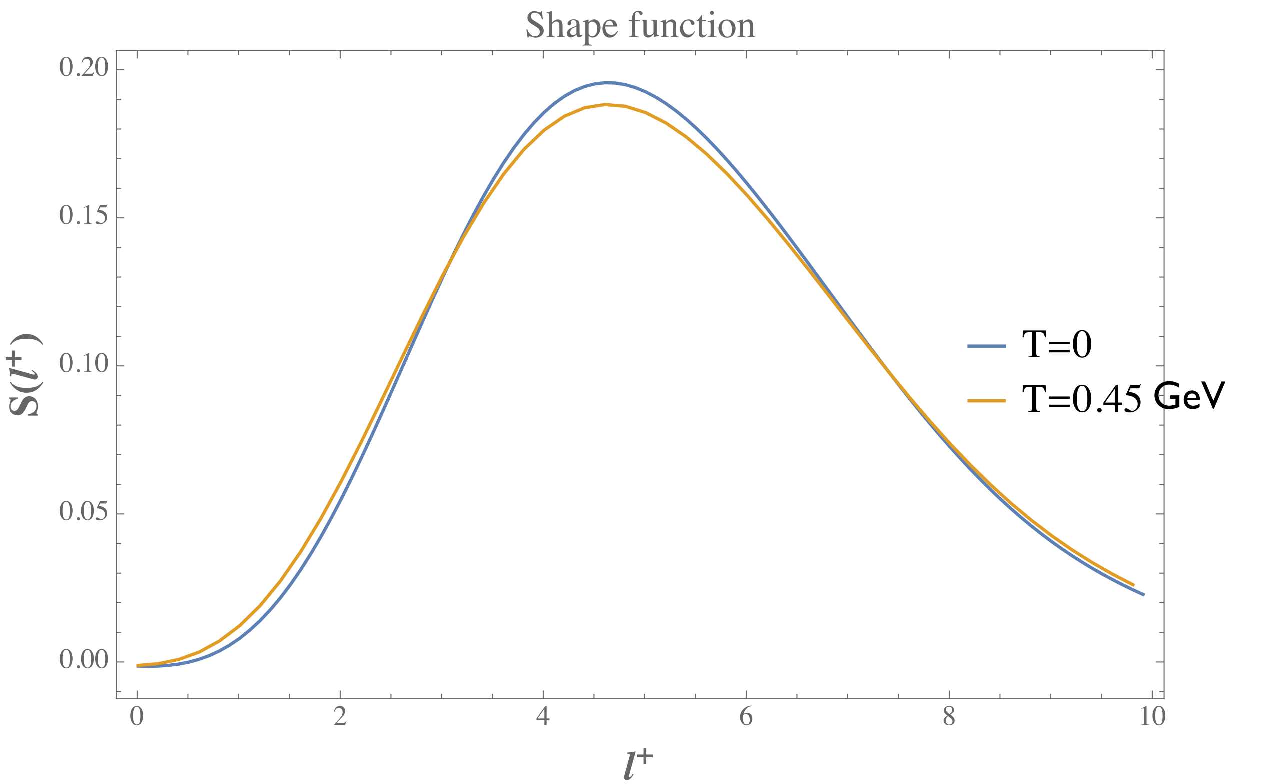
$$S(l_+) = \int \frac{\mathrm{d}\nu}{2\pi i} e^{\nu l^+} \langle e^{-\nu \, \mathbb{P}_\mathrm{R}} \rangle$$

$$\langle e^{-\nu \, \mathbb{P}_\mathrm{R}} \rangle = \exp \left\{ \frac{1}{2} \int_0^\infty \mathrm{d}\eta \int_0^{\mu^2} \mathrm{d}\boldsymbol{k}^2 \frac{\rho(\boldsymbol{k}^2)}{\coth \left[|\boldsymbol{k}| \cosh \eta/(2T) \right]} \left(e^{-\nu \, |\boldsymbol{k}| e^{-\eta} \coth \left[|\boldsymbol{k}| \cosh \eta/(2T) \right]} - 1 \right) \right\}$$

$$\lim_{l^+ \to 0} S(l^+) \simeq (l^+)^{a_\mathrm{PT}-1}$$

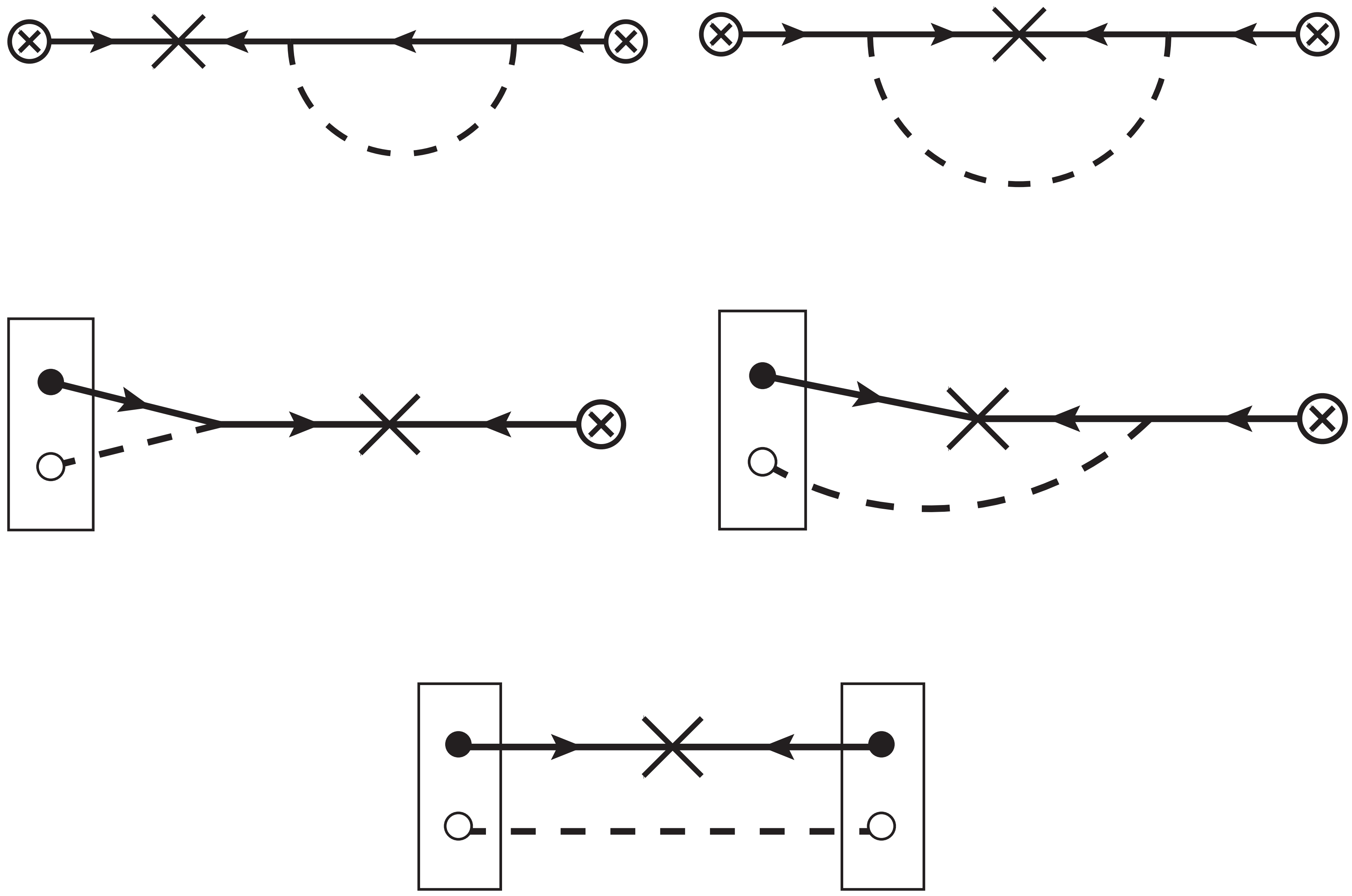
$$S^{(0)}(l^+) \simeq \delta\left(l^+ - \langle \mathbb{P}_\mathrm{R} \rangle\right)$$

- $\mathcal{O}(g^2)$: cumulants = moments
- 'tube'/string model with $a_{PT} \simeq 3$
  - boost-inv. rapidity density of gluons
  - NP gluon density not modified in medium!
- broadening of the distribution



Feynman "Photon hadron interactions" 1972
Webber hep-ph/9411384
KT, Casalderrey-Solana (2016, in preparation)

NLO CONTRIBUTIONS



KT, Casalderrey-Solana, Pablos (2016, in preparation)

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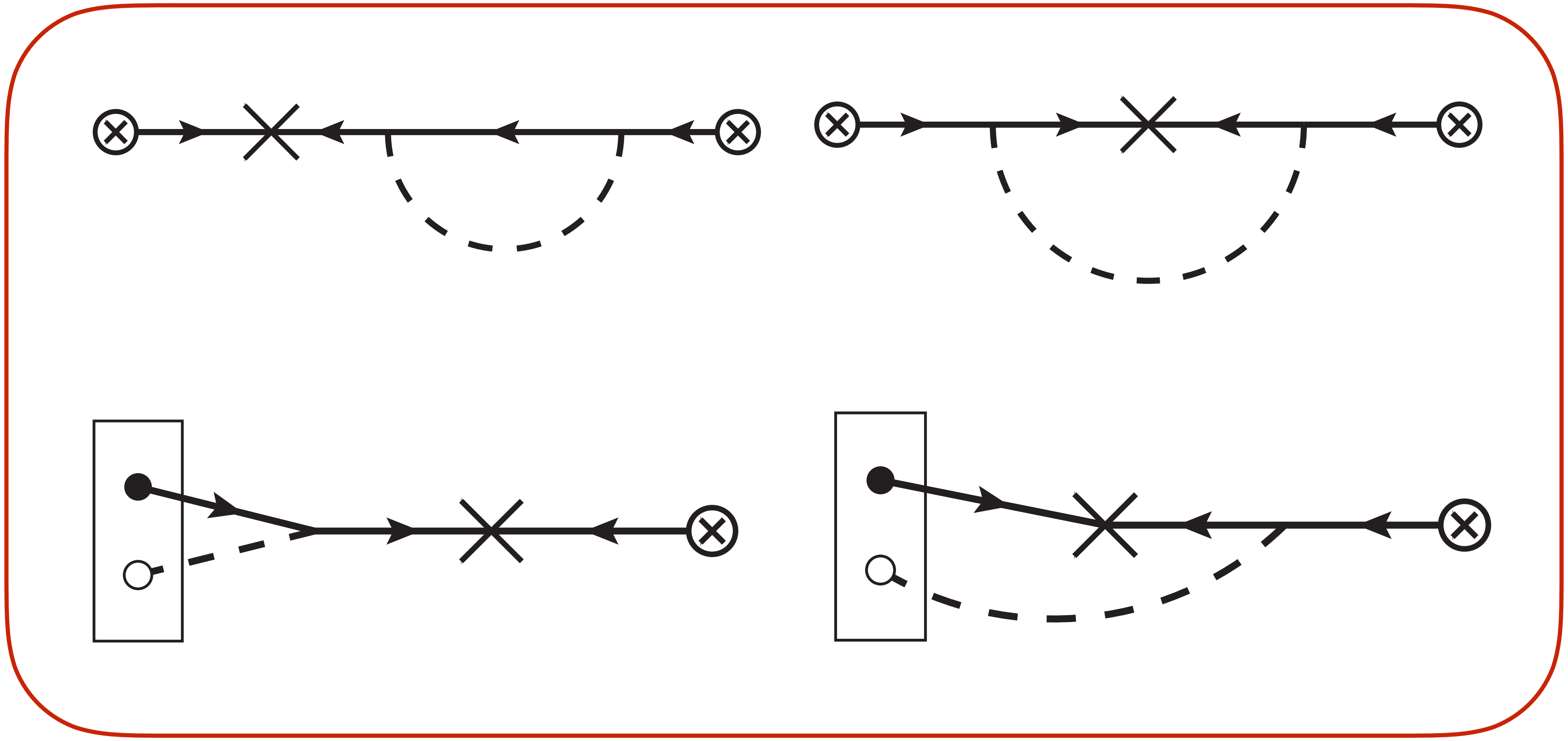

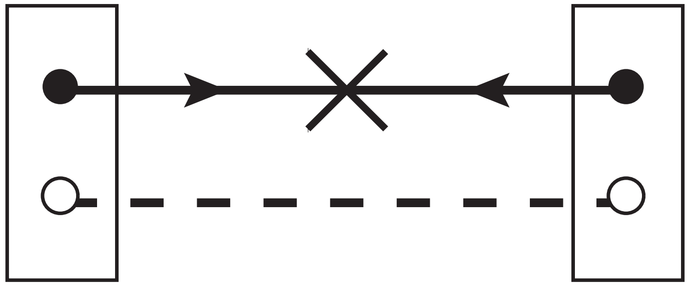

TFT: collinear divergences $\propto$ ***T***!

KT, Casalderrey-Solana, Pablos (2016, in preparation)

- collinear divergences cured by HTL resummation
  - particles acquire thermal mass: $\boldsymbol{k}^2 \to \boldsymbol{k}^2 + m_\infty^2$
  - matching of divergences provide UV regularisation for divergent HTL integrals
  - off-shell degrees of freedom: space-like correlation
- small-angle regime: LPM effect
  - novel power counting with $\mu_{\rm us}$
  - resummation (AMY)

KT, Casalderrey-Solana, Pablos (2016, in preparation)

Conclusions

- universal shape function in the vacuum
- clean theoretical setup: separation of jet and medium scales
- $\mathcal{O}(g^2)$:
  - on-shell contribution — temperature broadening
  - modification of the 'tube' model
- $\mathcal{O}(g^4)$:
  - sensitivity to in-medium dynamics
  - emergence of LPM physics: resummation
- application to other observables

BACK-UP

UNIVERSAL SHAPE FUNCTION

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma}{dt}\bigg|_{\rm PT} = \frac{dR_{\rm PT}(t)}{dt}\,, \qquad R_{\rm PT}(t) \overset{\rm DL}{=} \exp\left(-\frac{4\alpha_s(Q)}{3\pi}\ln^2 t\right)$$

scaling: $$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma}{dt}\bigg|_{\rm nonPT} = Q f(Qt)$$

$$\langle {\rm e}^{-\nu t}\rangle = \langle {\rm e}^{-\nu t}\rangle_{\rm PT} \times \int_0^\infty d\varepsilon\, {\rm e}^{-\nu\varepsilon/Q} f(\varepsilon;\mu)$$

$$R(t) = \int_0^{tQ} d\varepsilon\, f(\varepsilon;\mu) R_{\rm PT}\left(t - \frac{\varepsilon}{Q};\mu\right)$$

$$R(t) = R_{\rm PT}(t) - \frac{\langle\varepsilon\rangle}{Qt} R'_{\rm PT}(t) + \frac{\langle\varepsilon^2\rangle}{2(Qt)^2}\left[R''_{\rm PT}(t) - R'_{\rm PT}(t)\right] + \ldots$$

Catani, Trentadue, Turnock, Webber, NPB407 (1993) 3
Korchemsky hep-ph/9806537