Unintegrated double parton distributions

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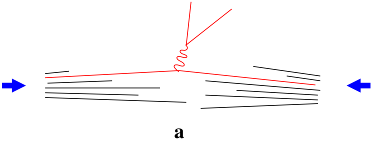
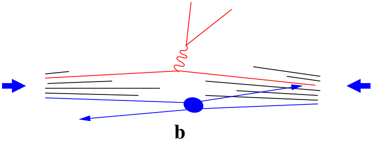
Plan

- Double parton scattering
- Evolution equations for double parton distributions
- Unintegrated single PDFs
- Unintegrated double PDFs
- Summary and outlook

- Hard processes due to collisions of quark and gluons: $pp \to X_{hard} + Y$



- A second hard interaction - double parton scattering (DPS).



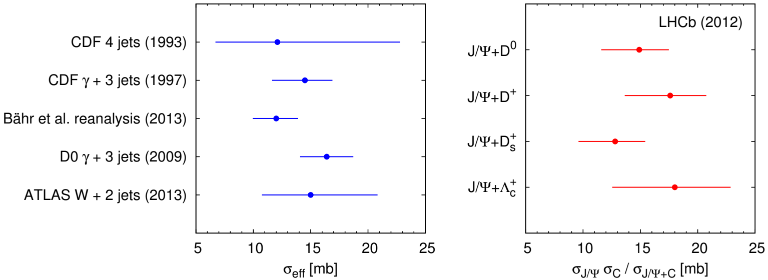
- Multiparton interactions - crucial for modeling of underlying event.

Experimental evidence

- Pocket formula

$$\sigma_{AB}^{\rm DPS} = \frac{\sigma_A^{\rm SPS}\,\sigma_B^{\rm SPS}}{\sigma_{\rm eff}}, \qquad\qquad \sigma_{\rm eff} \approx 15\ {\rm mb}$$

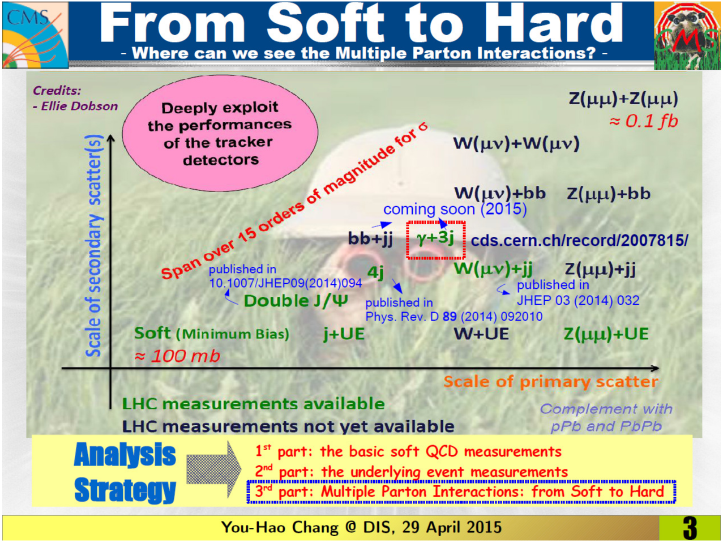
- Effective cross section



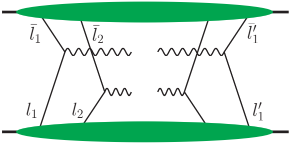
- Multiparton parton interactions are important at the LHC due to large parton flux.

(Review by M. Diehl, D. Ostermeier, A. Schafer, JHEP 1203 (2012) 089)

DPS at the LHC



- Two hard scatterings in one event with two hard scales, Q_1 and Q_2



- Collinear factorization cross section

  (Diehl,Gaunt,Ostermeier,Ploessl,Schäfer, JHEP 1601 (2016) 076, partial proof for DY)

$$\frac{d\sigma_{AB}^{DPS}}{dx_1\,dx_2\,dx_1'\,dx_2'} = \sum_{flav} \int d^2\mathbf{q}\, D_{f_1 f_2}(x_1, x_2, Q_1, Q_2, \mathbf{q})\, \sigma^A_{f_1 f_1'}(Q_1)\, \sigma^B_{f_2 f_2'}(Q_2)\, D_{f_1' f_2'}(x_1', x_2', Q_1, Q_2, -\mathbf{q})$$

- On shell matrix elements: $\sigma^A_{f_1 f_1'}$,and $\sigma^B_{f_2 f_2'}$ with incoming parton momenta $l_i^2 = 0$.

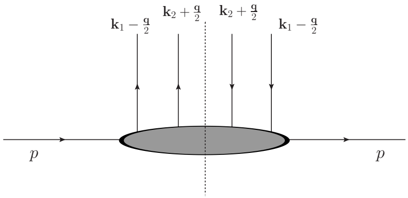
- Double parton distribution functions (DPDFs)

$$D_{f_1 f_2}(x_1, x_2, Q_1, Q_2, \mathbf{q})\,, \qquad x_1, x_2 > 0\,, \qquad x_1 + x_2 \leq 1$$

Double parton distribution functions

- Nonperturbative information about parton correlations in color (ab), spin $(\alpha\beta)$, flavor $(f_1 f_2)$ and momenta $k_i = x_i p + \mathbf{k}_i$ from unintegrated DPDFs

$$F^{(ab)}_{(\alpha\beta)(f_1 f_2)}(x_1, x_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{q})$$



- Transverse momentum structure

$$(\mathbf{k}_1 - \tfrac{1}{2}\mathbf{q}) + (\mathbf{k}_2 + \tfrac{1}{2}\mathbf{q}) = (\mathbf{k}_2 - \tfrac{1}{2}\mathbf{q}) + (\mathbf{k}_1 + \tfrac{1}{2}\mathbf{q})$$

- Color singlet, spin averaged, integrated DPDFs

$$D_{f_1 f_2}(x_1, x_2, \mathbf{q}) = \int d^2\mathbf{k}_1 d^2\mathbf{k}_2\, F_{f_1 f_2}(x_1, x_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{q})$$

Why unintegrated DPDFs?

- We want to unfold the unintegrated DPDFs to compute DPS cross sections with $k_\perp$-factorization

$$\frac{d\sigma_{AB}^{DPS}}{dx_1 dx_2 dx_1' dx_2'} = \int d^2 k_{1\perp} d^2 k_{2\perp} d^2 q \quad F_{f_1 f_2}(x_1, x_2, k_{1\perp}, k_{2\perp}, q, Q_1, Q_2)$$
$$\times\, \sigma^A_{f_1 f_1'}\, \sigma^B_{f_2 f_2'}\, F_{f_1' f_2'}(x_1', x_2', k_{1\perp}', k_{2\perp}', -q, Q_1, Q_2)$$

- Off-shell matrix elements: $\sigma^A_{f_1 f_1'}$ and $\sigma^B_{f_2 f_2'}$ with incoming parton momenta $l_i^2 \neq 0$.

- We want to go beyond a simplified approximation

$$f_{ab}(x_1, x_2, k_{1\perp}, k_{2\perp}, q = 0, Q_1, Q_2) \approx f_a(x_1, k_{1\perp}, Q_1)\, f_b(x_2, k_{2\perp}, Q_2)$$

(Kutak,Maciuła,Serino,Szczurek,van Hameren, JHEP 1604 (2016) 175, on 4-jet in DPS)

- DGLAP type evolution equation in LLA with nonhomogeneous splitting term

- For equal scales, $Q_1 = Q_2 \equiv Q$, and $\mathbf{q} = \mathbf{0}$

$$\frac{\partial}{\partial \ln Q^2} D_{f_1 f_2}(x_1, x_2, Q, Q) = \frac{\alpha_s}{2\pi} \sum_{f'} \Bigg\{ \int_{x_1}^{1-x_2} \frac{dz}{z} \mathcal{P}_{f_1 f'}\left(\frac{x_1}{z}\right) D_{f' f_2}(z, x_2, Q, Q)$$

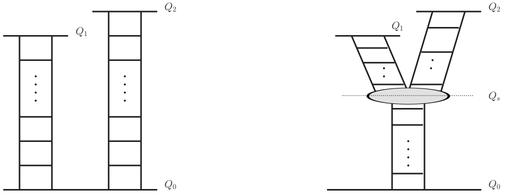
$$+ \int_{x_2}^{1-x_1} \frac{dz}{z} \mathcal{P}_{f_2 f'}\left(\frac{x_2}{z}\right) D_{f_1 f'}(x_1, z, Q, Q)$$

$$+ \mathcal{P}_{f' \to f_1 f_2}\left(\frac{x_1}{x_1 + x_2}\right) D_{f'}(x_1 + x_2, Q) \Bigg\}$$

(Konishi,Ukawa,Venziano,Snigirev,Zinovev,Shelest, ’80)

- The sum of homogeneous and non-homogeneous solutions, $Q_{min} = \min\{Q_1, Q_2\}$ (in matrix form in flavor space and in double Mellin moment representation)

$$\tilde{D}(n_1, n_2, Q_1, Q_2) = \tilde{E}(n_1, Q_1, Q_0)\, \tilde{D}(n_1, n_2, Q_0, Q_0)\, \tilde{E}^T(n_2, Q_2, Q_0)$$
$$+ \int_{Q_0^2}^{Q_{min}^2} \frac{dQ_s^2}{Q_s^2}\, \tilde{E}(n_1, Q_1, Q_s)\, \tilde{D}^{(sp)}(n_1, n_2, Q_s)\, \tilde{E}^T(n_2, Q_2, Q_s)$$



- where $\tilde{D}_{f_1 f_2}(n_1, n_2, Q_0, Q_0)$ are initial conditions and splitting distributions

$$\tilde{D}^{(sp)}_{f_1 f_2}(n_1, n_2, Q_s) = \frac{\alpha_s(Q_s)}{2\pi} \sum_f \underbrace{\tilde{D}_f(n_1 + n_2, Q_s)}_{single\ \ PDFs} \int_0^1 dz z^{n_1} (1-z)^{n_2} P_{f \to f_1 f_2}(z)$$

Evolution functions $E_{ff'}$

- $E_{ff'}$ evolve single PDFs from the scale $Q_0 \to Q$

$$\tilde{D}_f(n,Q) = \sum_{f'} \tilde{E}_{ff'}(n,Q,Q_0)\,\tilde{D}_{f'}(n,Q_0)$$

- Obey DGLAP equations with initial condition $E_{ff'}(n,Q_0,Q_0) = \delta_{ff'}$

$$\frac{\partial}{\partial \ln Q^2}\tilde{E}_{ff'}(n,Q,Q_0) = \underbrace{\sum_h \tilde{P}_{fh}(n,Q)\tilde{E}_{hf'}(n,Q,Q_0)}_{real\ \ emission}$$

$$- \tilde{E}_{ff'}(n,Q,Q_0)\underbrace{\sum_h \int_0^1 dz z P_{hf}(z,Q)}_{virtual\ \ corrections}$$

- which can be transformed into the integral equation:

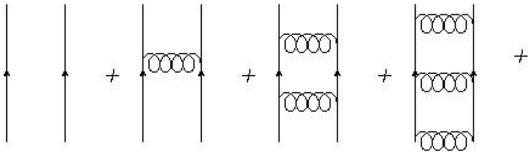
Interpretation of master equation

$$\tilde{E}_{ff'}(n,Q,Q_0) = T_f(Q,Q_0)\,\delta_{ff'} + \int_{Q_0^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\, T_f(Q,k_\perp) \sum_h \tilde{P}_{fh}(n,k_\perp)\,\tilde{E}_{hf'}(n,k_\perp,Q_0)$$

- Real parton emissions interrupted by evolution with Sudakov form factor

$$T_f(Q,k_\perp) = \exp\left\{ -\int_{k_\perp^2}^{Q^2} \frac{dp_\perp^2}{p_\perp^2} \sum_h \int_0^{1-\Delta} dzzP_{hf}(z,k_\perp) \right\}$$

- $E_{ff'}$ is an infinite sum of parton ladders



Unintegrated single PDFs

- Pluging the master equation into

$$D_a(n,Q) = \sum_b E_{ab}(n,Q,Q_0)\, D_b(n,Q_0)$$

one finds in the x-space

$$D_f(x,Q) = T_f(Q,Q_0) D_f(x,Q_0) + \int_{Q_0^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\, F_f(x,k_\perp,Q)$$

- where F_f are unintegrated PDFs for $k_\perp \geq Q_0$

$$F_f(x,k_\perp,Q) = T_f(Q,k_\perp) \sum_h \int_x^{1-\Delta} \frac{dz}{z}\, P_{fh}(z,k_\perp)\, D_h\Big(\frac{x}{z},k_\perp\Big)$$

- with the cutoff corresponding to DGLAP or CCFM ordering (Kimber, Martin and Ryskin, Eur.Phys.J. C12 (2000) 655, Phys.Rev. D63 (2001) 114027)

$$\Delta = \frac{k_\perp}{Q} \qquad \text{or} \qquad \Delta = \frac{k_\perp}{k_\perp + Q}$$

- Transverse momenta below Q_0 are integrated out to $D_f(x,Q_0)$.

Construction of unintegrated DPDFs

- Substitute the master equation to the solution of the evolution equations for DPDFs

$$\tilde{D}(n_1,n_2,Q_1,Q_2) = \underbrace{\tilde{E}(n_1,Q_1,Q_0)\,\tilde{D}(n_1,n_2,Q_0,Q_0)\,\tilde{E}^T(n_2,Q_2,Q_0)}_{homogeneous}$$

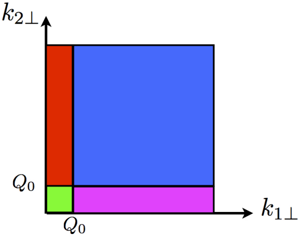
$$+ \underbrace{\int_{Q_0^2}^{Q^2_{min}} \frac{dQ'^2}{Q'^2}\, E(n_1,Q_1,Q')\, D^{(sp)}(n_1,n_2,Q')\, E^T(n_2,Q_2,Q')}_{non-homogeneous}$$

- Homogeneous and non-homogeneous UDPDFs from the above sum

$$\tilde{F}_{f_1 f_2}(n_1,n_2,k_{1\perp},k_{2\perp},Q_1,Q_2) = \tilde{F}^{(H)}_{f_1 f_2} + \tilde{F}^{(NH)}_{f_1 f_2}$$

Homogeneous unintegrated DPDFs

- Four regions of transverse momenta



- Non-perturbative: $k_{1\perp}, k_{2\perp} < Q_0$
- Half-perturbative: $k_{1\perp} \geq Q_0 \quad k_{2\perp} < Q_0$
- Half-perturbative: $k_{1\perp} < Q_0 \quad k_{2\perp} \geq Q_0$
- Perturbative: $k_{1\perp}, k_{2\perp} \geq Q_0$

- In each region different prescription for unintegrated DPDFs.

$$\tilde{D}^{(H)}_{f_1 f_2}(n_1,n_2,Q_1,Q_2) = T_{f_1}(Q_1,Q_0)\,T_{f_2}(Q_2,Q_0)\,\tilde{D}_{f_1 f_2}(n_1,n_2,Q_0,Q_0)$$
$$+\int_{Q_0^2}^{Q_2^2}\frac{dk_{2\perp}^2}{k_{2\perp}^2}\,T_{f_1}(Q_1,Q_0)\,T_{f_2}(Q_2,k_{2\perp})\sum_h \tilde{P}_{f_2 h}(n_2,k_{2\perp})\,\tilde{D}^{(H)}_{f_1 h}(n_1,n_2,Q_0,k_{2\perp})$$
$$+\int_{Q_0^2}^{Q_1^2}\frac{dk_{1\perp}^2}{k_{1\perp}^2}\,T_{f_1}(Q_1,k_{1\perp})\,T_{f_2}(Q_2,Q_0)\sum_h \tilde{P}_{f_1 h}(n_1,k_{1\perp})\,\tilde{D}^{(H)}_{h f_2}(n_1,n_2,k_{1\perp},Q_0)$$
$$+\int_{Q_0^2}^{Q_1^2}\frac{dk_{1\perp}^2}{k_{1\perp}^2}\int_{Q_0^2}^{Q_2^2}\frac{dk_{2\perp}^2}{k_{2\perp}^2}\,T_{f_1}(Q_1,k_{1\perp})\,T_{f_2}(Q_2,k_{2\perp})$$
$$\times\sum_{hh'}\tilde{P}_{f_1 h}(n_1,k_{1\perp})\,\tilde{P}_{f_2 h'}(n_2,k_{2\perp})\,\tilde{D}^{(H)}_{hh'}(n_1,n_2,k_{1\perp},k_{2\perp})$$

- In the first three formulas at least one transverse momentum is integrated out in the non-perturbative region $0 < k_{i\perp} < Q_0$. In the corresponding off-shell matrix element we set $k_{i\perp} = 0$.

Homogeneous unintegrated DPDFs in x-space

- In the perturbative region, $(k_{1\perp}, k_{2\perp} \geq Q_0)$, we have the unintegrated DPDFs

$$F^{(H)}_{f_1 f_2}(x_1, x_2, k_{1\perp}, k_{2\perp}, Q_1, Q_2) = T_{f_1}(Q_1, k_{1\perp})\, T_{f_2}(Q_2, k_{2\perp})$$
$$\times \sum_{hh'} \int_{\frac{x_1}{1-x_2}}^{1-\Delta_1} \frac{dz_1}{z_1} \int_{\frac{x_2}{1-x_1/z_1}}^{1-\Delta_2} \frac{dz_2}{z_2} P_{f_1 h}(z_1, k_{1\perp})\, P_{f_2 h'}(z_2, k_{2\perp})\, D^{(H)}_{hh'}\left(\frac{x_1}{z_1}, \frac{x_2}{z_2}, k_{1\perp}, k_{2\perp}\right)$$

with the cut-off parameters

$$\Delta_i = \frac{k_{i\perp}}{Q_i} \qquad \text{or} \qquad \Delta_i = \frac{k_{i\perp}}{k_{i\perp} + Q_i}, \qquad i = 1, 2$$

- Transverse momenta from unfolding the last step in the evolution of two ladders.

- Similar formulas for unintegrated DPDFs in half-perturbative regions.

Non-homogeneous UPDFs

- The same treatment - insert the master equation

$$\tilde{E}_{ab}(n,Q,Q_0) = T_a(Q,Q_0)\,\delta_{ab} + \int_{Q_0^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\, T_a(Q,k_\perp) \sum_{a'} \tilde{P}_{aa'}(n,k_\perp)\, \tilde{E}_{a'b}(n,k_\perp,Q_0)$$

into the non-homogeneous part of DPDFs

$$\tilde{D}^{(nh)}(n_1,n_2,Q_1,Q_2) = \int_{Q_0^2}^{Q_{min}^2} \frac{dQ_s^2}{Q_s^2}\, \tilde{E}(n_1,Q_1,Q_s)\, D^{(sp)}(n_1,n_2,Q_s)\, \tilde{E}^T(n_2,Q_2,Q_s)$$

- Transverse momenta can also be generated by unfolding $D^{(sp)}(n_1,n_2,Q_s)$.
- Details in the forthcoming paper.

Summary and outlook

- Starting from the integrated DPDFs, we constructed the unintegrated DPDFs by unfolding the last step of the evolution - KMR approach.
- The construction of the homogeneous part of UPDFs is rather straightforward but the non-homogeneous part is more subtle.
- The homogeneous UDPDFs have nontrivial correlations between longitudinal and transverse momenta, e.g

$$\frac{x_1}{1-\Delta_1} + \frac{x_2}{1-\Delta_2} \le 1 \qquad (\Delta_i = k_{i\perp}/Q_i)$$

- Numerics is challenging but possible.

Backup slides

Initial conditions

(GB, Lewandowska,Serino,Snyder,Staśto, PLB 750 (2015) 559)

- Usual assumption at Q_0

$$D_{ab}(x_1, x_2) = D_a(x_1)\, D_b(x_2)\, \rho(x_1, x_2)$$

- but such DPDFs do not fulfill sum rules

$$\sum_a \int_0^{1-x_2} dx_1 x_1\, D_{ab}(x_1, x_2) \;=\; (1 - x_2) D_b(x_2)$$

$$\int_0^{1-x_2} dx_1 \left\{ D_{qb}(x_1, x_2) - D_{\bar{q}b}(x_1, x_2) \right\} \;=\; (N_q - \delta_{qb} + \delta_{\bar{q}b})\, D_b(x_2)$$

- In pure gluon case we constructed such distribution using MSTW08 parameters

$$D_{gg}(x_1, x_2) = \sum_{k=1}^{3} A_k\, (x_1\, x_2)^{\alpha_k}\, (1 - x_1 - x_2)^{\eta - \alpha_k - 1} \;\neq\; D_g(x_1) D_g(x_2)$$

- Full case leads to negative DPDFs at large x.

Numerical comparison