Unintegrated double parton distributions

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Various Faces of QCD, Świerk, 8th October 2016
Plan

- Double parton scattering
- Evolution equations for double parton distributions
- Unintegrated single PDFs
- Unintegrated double PDFs
- Summary and outlook
Introduction

- Hard processes due to collisions of quark and gluons: \( pp \rightarrow X_{\text{hard}} + Y \)

- A second hard interaction - double parton scattering (DPS).

- Multiparton interactions - crucial for modeling of underlying event.
Experimental evidence

▶ Pocket formula

\[
\sigma_{AB}^{DPS} = \frac{\sigma_A^{SPS} \sigma_B^{SPS}}{\sigma_{eff}}, \quad \sigma_{eff} \approx 15 \text{ mb}
\]

▶ Effective cross section

Multiparton parton interactions are important at the LHC due to large parton flux.

DPS at the LHC

From Soft to Hard
- Where can we see the Multiple Parton Interactions? -

Deeply exploit the performances of the tracker detectors

Scale of secondary scatter(s)

Span over 15 orders of magnitude for $\sigma$

LHC measurements available
LHC measurements not yet available

Analysis Strategy

1st part: the basic soft QCD measurements
2nd part: the underlying event measurements
3rd part: Multiple Parton Interactions: from Soft to Hard

You-Hao Chang @ DIS, 29 April 2015
Double parton scattering

- Two hard scatterings in one event with two hard scales, $Q_1$ and $Q_2$

- Collinear factorization cross section
  
  \[
  \frac{d\sigma_{AB}^{DPS}}{dx_1 dx_2 dx'_1 dx'_2} = \sum_{\text{flav}} \int d^2 q \ D_{f_1 f_2}(x_1, x_2, Q_1, Q_2, q) \ \sigma_{f_1 f'_1}^A(Q_1) \ \sigma_{f_2 f'_2}^B(Q_2) \ D_{f'_1 f'_2}(x'_1, x'_2, Q_1, Q_2, -q)
  \]

- On shell matrix elements: $\sigma_{f_1 f'_1}^A$ and $\sigma_{f_2 f'_2}^B$ with incoming parton momenta $l_i^2 = 0$.

- Double parton distribution functions (DPDFs)

\[
D_{f_1 f_2}(x_1, x_2, Q_1, Q_2, q), \quad x_1, x_2 > 0, \quad x_1 + x_2 \leq 1
\]
Double parton distribution functions

- Nonperturbative information about parton correlations in color \((ab)\), spin \((\alpha\beta)\), flavor \((f_1 f_2)\) and momenta \(k_i = x_i p + k_i\) from unintegrated DPDFs

\[ F^{(ab)}_{(\alpha\beta)(f_1 f_2)}(x_1, x_2, k_1, k_2, q) \]

- Transverse momentum structure

\[(k_1 - \frac{1}{2}q) + (k_2 + \frac{1}{2}q) = (k_2 - \frac{1}{2}q) + (k_1 + \frac{1}{2}q)\]

- Color singlet, spin averaged, integrated DPDFs

\[ D_{f_1 f_2}(x_1, x_2, q) = \int d^2 k_1 d^2 k_2 \, F_{f_1 f_2}(x_1, x_2, k_1, k_2, q) \]
Why unintegrated DPDFs?

▶ We want to unfold the unintegrated DPDFs to compute DPS cross sections with $k_\perp$-factorization

$$\frac{d\sigma^{DPS}_{AB}}{dx_1 dx_2 dx'_1 dx'_2} = \int d^2 k_1 \perp d^2 k_2 \perp d^2 q \quad F_{f_1 f_2}(x_1, x_2, k_1 \perp, k_2 \perp, q, Q_1, Q_2)$$

$$\times \sigma_{f_1 f'_1}^A \sigma_{f_2 f'_2}^B F_{f'_1 f'_2}(x'_1, x'_2, k'_1 \perp, k'_2 \perp, -q, Q_1, Q_2)$$

▶ Off-shell matrix elements: $\sigma_{f_1 f'_1}^A$ and $\sigma_{f_2 f'_2}^B$ with incoming parton momenta $l_i^2 \neq 0$.

▶ We want to go beyond a simplified approximation

$$f_{ab}(x_1, x_2, k_1 \perp, k_2 \perp, q = 0, Q_1, Q_2) \approx f_a(x_1, k_1 \perp, Q_1) f_b(x_2, k_2 \perp, Q_2)$$

(Kutak, Maciulà, Serino, Szczurek, van Hameren, JHEP 1604 (2016) 175, on 4-jet in DPS)
DGLAP type evolution equation in LLA with nonhomogeneous splitting term

For equal scales, \( Q_1 = Q_2 \equiv Q \), and \( q = 0 \)

\[
\frac{\partial}{\partial \ln Q^2} D_{f_1 f_2}(x_1, x_2, Q, Q) = \frac{\alpha_s}{2\pi} \sum_{f'} \left\{ \int_{x_1}^{1-x_2} \frac{dz}{z} \mathcal{P}_{f_1 f'} \left( \frac{x_1}{z} \right) D_{f' f_2}(z, x_2, Q, Q) + \int_{x_2}^{1-x_1} \frac{dz}{z} \mathcal{P}_{f_2 f'} \left( \frac{x_2}{z} \right) D_{f_1 f'}(x_1, z, Q, Q) + \mathcal{P}_{f' \rightarrow f_1 f_2} \left( \frac{x_1}{x_1 + x_2} \right) D_{f'}(x_1 + x_2, Q) \right\}
\]

(Konishi, Ukawa, Venziano, Snigirev, Zinovev, Shelest, '80)
Solution to evolution equations for DPDFs

- The sum of **homogeneous** and **non-homogeneous** solutions, $Q_{\text{min}} = \min\{Q_1, Q_2\}$ (in matrix form in flavor space and in double Mellin moment representation)

$$
\tilde{D}(n_1, n_2, Q_1, Q_2) = \tilde{E}(n_1, Q_1, Q_0) \tilde{D}(n_1, n_2, Q_0, Q_0) \tilde{E}^T(n_2, Q_2, Q_0)
$$

$$
+ \int_{Q_0^2}^{Q_{\text{min}}^2} \frac{dQ_s^2}{Q_s^2} \tilde{E}(n_1, Q_1, Q_s) \tilde{D}^{(sp)}(n_1, n_2, Q_s) \tilde{E}^T(n_2, Q_2, Q_s)
$$

- where $\tilde{D}_{f_1f_2}(n_1, n_2, Q_0, Q_0)$ are initial conditions and splitting distributions

$$
\tilde{D}_{f_1f_2}^{(sp)}(n_1, n_2, Q_s) = \frac{\alpha_s(Q_s)}{2\pi} \sum_f \tilde{D}_f(n_1 + n_2, Q_s) \int_0^1 dzz^{n_1}(1-z)^{n_2} P_{f \rightarrow f_1f_2}(z)
$$
Evolution functions $E_{ff'}$

- $E_{ff'}$ evolve single PDFs from the scale $Q_0 \rightarrow Q$

$$\tilde{D}_f(n, Q) = \sum_{f'} \tilde{E}_{ff'}(n, Q, Q_0) \tilde{D}_{f'}(n, Q_0)$$

- Obey DGLAP equations with initial condition $E_{ff'}(n, Q_0, Q_0) = \delta_{ff'}$

$$\frac{\partial}{\partial \ln Q^2} \tilde{E}_{ff'}(n, Q, Q_0) = \sum_{h} \tilde{P}_{fh}(n, Q) \tilde{E}_{hf'}(n, Q, Q_0)$$

real emission

$$- \tilde{E}_{ff'}(n, Q, Q_0) \sum_{h} \int_{0}^{1} dzzP_{hf}(z, Q)$$

virtual corrections

- which can be transformed into the integral equation:
\[ \bar{E}_{ff'}(n, Q, Q_0) = T_f(Q, Q_0) \delta_{ff'} + \int_{Q_0^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} T_f(Q, k_{\perp}) \sum_h \bar{P}_{fh}(n, k_{\perp}) \bar{E}_{hf'}(n, k_{\perp}, Q_0) \]

- Real parton emissions interrupted by evolution with Sudakov form factor
  \[ T_f(Q, k_{\perp}) = \exp \left\{ - \int_{k_{\perp}^2}^{Q^2} \frac{dp_{\perp}^2}{p_{\perp}^2} \sum_h \int_0^{1-\Delta} dzzP_{hf}(z, k_{\perp}) \right\} \]

- \( E_{ff'} \) is an infinite sum of parton ladders

\[ \text{\textcopyright{\hspace{1cm}}} \hspace{1cm} \text{\textcopyright{\hspace{1cm}}} \hspace{1cm} \]
Plugging the master equation into

\[ D_a(n, Q) = \sum_b E_{ab}(n, Q, Q_0) D_b(n, Q_0) \]

one finds in the \( x \)-space

\[ D_f(x, Q) = T_f(Q, Q_0) D_f(x, Q_0) + \int_{Q_0^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} F_f(x, k_{\perp}, Q) \]

where \( F_f \) are unintegrated PDFs for \( k_{\perp} \geq Q_0 \)

\[ F_f(x, k_{\perp}, Q) = T_f(Q, k_{\perp}) \sum_h \int_x^{1-\Delta} \frac{dz}{z} P_{fh}(z, k_{\perp}) D_h \left( \frac{x}{z}, k_{\perp} \right) \]


\[ \Delta = \frac{k_{\perp}}{Q} \quad \text{or} \quad \Delta = \frac{k_{\perp}}{k_{\perp} + Q} \]

Transverse momenta below \( Q_0 \) are integrated out to \( D_f(x, Q_0) \).
Substitute the master equation to the solution of the evolution equations for DPDFs

\[ \tilde{D}(n_1, n_2, Q_1, Q_2) = \tilde{E}(n_1, Q_1, Q_0) \tilde{D}(n_1, n_2, Q_0, Q_0) \tilde{E}^T(n_2, Q_2, Q_0) \]

\[ + \int_{Q_0^2}^{Q_{\text{min}}^2} \frac{dQ'^2}{Q'^2} E(n_1, Q_1, Q') D^{(sp)}(n_1, n_2, Q') E^T(n_2, Q_2, Q') \]

Homogeneous and non-homogeneous UDPDFs from the above sum

\[ \tilde{F}_{f_1 f_2} (n_1, n_2, k_{1\perp}, k_{2\perp}, Q_1, Q_2) = \tilde{F}_{f_1 f_2}^{(H)} + \tilde{F}_{f_1 f_2}^{(NH)} \]
Homogeneous unintegrated DPDFs

- Four regions of transverse momenta

- Non-perturbative: \( k_1 \perp, k_2 \perp < Q_0 \)
- Half-perturbative: \( k_1 \perp \geq Q_0 \), \( k_2 \perp < Q_0 \)
- Half-perturbative: \( k_1 \perp < Q_0 \), \( k_2 \perp \geq Q_0 \)
- Perturbative: \( k_1 \perp, k_2 \perp \geq Q_0 \)

- In each region different prescription for unintegrated DPDFs.
Homogeneous unintegrated DPDFs

\[ \tilde{D}_{f_1 f_2}^{(H)}(n_1, n_2, Q_1, Q_2) = T_{f_1}(Q_1, Q_0) \ T_{f_2}(Q_2, Q_0) \ \tilde{D}_{f_1 f_2}(n_1, n_2, Q_0, Q_0) \]

\[ + \int_{Q_0^2}^{Q_2^2} \frac{d k_{2\perp}^2}{k_{2\perp}^2} \ T_{f_1}(Q_1, Q_0) \ T_{f_2}(Q_2, k_{2\perp}) \sum_h \tilde{P}_{f_2 h}(n_2, k_{2\perp}) \ \tilde{D}_{f_1 h}(n_1, n_2, Q_0, k_{2\perp}) \]

\[ + \int_{Q_0^2}^{Q_1^2} \frac{d k_{1\perp}^2}{k_{1\perp}^2} \ T_{f_1}(Q_1, k_{1\perp}) \ T_{f_2}(Q_2, Q_0) \sum_h \tilde{P}_{f_1 h}(n_1, k_{1\perp}) \ \tilde{D}_{h f_2}(n_1, n_2, k_{1\perp}, Q_0) \]

\[ + \int_{Q_0^2}^{Q_1^2} \frac{d k_{1\perp}^2}{k_{1\perp}^2} \int_{Q_0^2}^{Q_2^2} \frac{d k_{2\perp}^2}{k_{2\perp}^2} \ T_{f_1}(Q_1, k_{1\perp}) \ T_{f_2}(Q_2, k_{2\perp}) \]

\[ \times \sum_{hh'} \tilde{P}_{f_1 h}(n_1, k_{1\perp}) \ \tilde{P}_{f_2 h'}(n_2, k_{2\perp}) \ \tilde{D}_{h h'}^{(H)}(n_1, n_2, k_{1\perp}, k_{2\perp}) \]

- In the first three formulas at least one transverse momentum is integrated out in the non-perturbative region \( 0 < k_{i\perp} < Q_0 \). In the corresponding off-shell matrix element we set \( k_{i\perp} = 0 \).
In the perturbative region, \((k_1 \perp, k_2 \perp \geq Q_0)\), we have the unintegrated DPDFs

\[
F_{f_1 f_2}^{(H)}(x_1, x_2, k_1 \perp, k_2 \perp, Q_1, Q_2) = T_{f_1}(Q_1, k_1 \perp) \ T_{f_2}(Q_2, k_2 \perp)
\times \sum_{hh'} 1-\Delta_1 \int \frac{dz_1}{z_1} \int \frac{dz_2}{z_2} P_{f_1 h}(z_1, k_1 \perp) \ P_{f_2 h'}(z_2, k_2 \perp) \ D_{hh'}^{(H)}\left(\frac{x_1}{z_1}, \frac{x_2}{z_2}, k_1 \perp, k_2 \perp\right)
\]

with the cut-off parameters

\[
\Delta_i = \frac{k_i \perp}{Q_i} \quad \text{or} \quad \Delta_i = \frac{k_i \perp}{k_i \perp + Q_i}, \quad i = 1, 2
\]

Transverse momenta from unfolding the last step in the evolution of two ladders.

Similar formulas for unintegrated DPDFs in half-perturbative regions.
Non-homogeneous UPDFs

The same treatment - insert the master equation

\[
\tilde{E}_{ab}(n, Q, Q_0) = T_a(Q, Q_0) \delta_{ab} + \int_{Q_0^2}^{Q^2} \frac{d k_\perp^2}{k_\perp^2} \ T_a(Q, k_\perp) \sum_{a'} \tilde{P}_{aa'}(n, k_\perp) \tilde{E}_{a'b}(n, k_\perp, Q_0)
\]

into the non-homogeneous part of DPDFs

\[
\tilde{D}^{(nh)}(n_1, n_2, Q_1, Q_2) = \int_{Q_0^2}^{Q_{\text{min}}^2} \frac{d Q_s^2}{Q_s^2} \tilde{E}(n_1, Q_1, Q_s) \ D^{(sp)}(n_1, n_2, Q_s) \tilde{E}^T(n_2, Q_2, Q_s)
\]

Transverse momenta can also be generated by unfolding \(D^{(sp)}(n_1, n_2, Q_s)\).

Details in the forthcoming paper.
Starting from the integrated DPDFs, we constructed the unintegrated DPDFs by unfolding the last step of the evolution - KMR approach.

The construction of the homogeneous part of UPDFs is rather straightforward but the non-homogeneous part is more subtle.

The homogeneous UDPDFs have nontrivial correlations between longitudinal and transverse momenta, e.g

\[
\frac{x_1}{1 - \Delta_1} + \frac{x_2}{1 - \Delta_2} \leq 1 \quad (\Delta_i = k_{i\perp}/Q_i)
\]

Numerics is challenging but possible.
Backup slides
Initial conditions

(GB, Lewandowska, Serino, Snyder, Staśto, PLB 750 (2015) 559)

▶ Usual assumption at $Q_0$

$$D_{ab}(x_1, x_2) = D_a(x_1) D_b(x_2) \rho(x_1, x_2)$$

▶ but such DPDFs do not fulfill sum rules

$$\sum_a \int_0^{1-x_2} dx_1 x_1 \, D_{ab}(x_1, x_2) = (1-x_2) D_b(x_2)$$

$$\int_0^{1-x_2} \left\{ D_{q_b}(x_1, x_2) - D_{\bar{q}_b}(x_1, x_2) \right\} = (N_q - \delta_{qb} + \delta_{\bar{q}b}) D_b(x_2)$$

▶ In pure gluon case we constructed such distribution using MSTW08 parameters

$$D_{gg}(x_1, x_2) = \sum_{k=1}^3 A_k (x_1 x_2)^{\alpha_k} (1-x_1-x_2)^{\eta-\alpha_k-1} \neq D_g(x_1) D_g(x_2)$$

▶ Full case leads to negative DPDFs at large $x$. 
Numerical comparison

\[ x_2 = 0.01 \]

\[ x_1 x_2 D_{gg} Q^2 = 1 \text{ GeV}^2 \]

\[ x_1 x_2 D_{gg} Q^2 = 10 \text{ GeV}^2 \]